XVII. *On* Fresnel's *Theory of the Aberration of Light.* *By*

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THE theory of the aberration of light, and of the absence of any influence of the motion of the earth on the laws of refraction, &c., given by Fresnel in the ninth volume of the *Annales de Chimie*, p. 57, is really very remarkable. If we suppose the diminished velocity of propagation of light within refracting media to arise solely from the greater density of the æther within them, the elastic force being the same as without, the density which it is necessary to suppose the æther within a medium of refractive index μ to have is μ^2, the density in vacuum being taken for unity. Fresnel supposes that the earth passes through the æther without disturbing it, the æther penetrating the earth quite freely. He supposes that a refracting medium moving with the earth carries with it a quantity of æther, of density μ^2-1, which constitutes the excess of density of the æther within it over the density of the æther in vacuum. He supposes that light is propagated through this æther, of which part is moving with the earth, and part is at rest in space, as it would be if the whole were moving with the velocity of the centre of gravity of any portion of it, that is, with a velocity $\left(1-\frac{1}{\mu^2}\right)v$, v being the velocity of the earth. It may be observed however that the result would be the same if we supposed the whole of the æther within the earth to move together, the æther entering the earth in front, and being immediately condensed, and issuing from it behind, where it is immediately rarefied, undergoing likewise sudden condensation or rarefaction in passing from one refracting medium to another. On this supposition, the evident condition that a mass v of the æther must pass in a unit of time across a plane of area unity, drawn anywhere within the earth in a direction perpendicular to that of the earth's motion, gives $\left(1-\frac{1}{\mu^2}\right)v$ for the velocity of the æther within a refracting medium. As this idea is rather simpler than Fresnel's, I shall adopt it in considering his theory. Also, instead of considering the earth as in motion and the æther outside it as at rest, it will be simpler to conceive a velocity equal and opposite to that of the earth impressed both on the earth and on the æther. On this supposition the earth will be at rest; the æther outside it will be moving with a velocity v, and the æther in a refracting medium with a velocity

* Communicated by the Author.

$\frac{v}{\mu^2}$, in a direction contrary to that of the earth's real motion. On account of the smallness of the coefficient of aberration, we may also neglect the square of the ratio of the earth's velocity to that of light; and if we resolve the earth's velocity in different directions, we may consider the effect of each resolved part separately.

In the ninth volume of the *Comptes Rendus* of the Academy of Sciences, p. 774, there is a short notice of a memoir by M. Babinet, giving an account of an experiment which seemed to present a difficulty in its explanation. M. Babinet found that when two pieces of glass of equal thickness were placed across two streams of light which interfered and exhibited fringes, in such a manner that one piece was traversed by the light in the direction of the earth's motion, and the other in the contrary direction, the fringes were not in the least displaced. This result, as M. Babinet asserts, is contrary to the theory of aberration contained in a memoir read by him before the Academy in 1829, as well as to the other received theories on the subject. I have not been able to meet with this memoir, but it is easy to show that the result of M. Babinet's experiment is in perfect accordance with Fresnel's theory.

Let T be the thickness of one of the glass plates, V the velocity of propagation of light in vacuum, supposing the æther at rest. Then $\frac{V}{\mu}$ would be the velocity with which light would traverse the glass if the æther were at rest; but the æther moving with a velocity $\frac{v}{\mu^2}$, the light traverses the glass with a velocity $\frac{V}{\mu} \pm \frac{v}{\mu^2}$, and therefore in a time

$$T \div \left(\frac{V}{\mu} \pm \frac{v}{\mu^2}\right) = \frac{\mu T}{V}\left(1 \mp \frac{v}{\mu V}\right).$$

But if the glass were away, the light, travelling with a velocity $V \pm v$, would pass over the space T in the time

$$T \div (V \pm v) = \frac{T}{V}\left(1 \mp \frac{v}{V}\right).$$

Hence the retardation, expressed in time, $= (\mu - 1)\frac{T}{V}$, the same as if the earth were at rest. But in this case no effect would be produced on the fringes, and therefore none will be produced in the actual case.

I shall now show that, according to Fresnel's theory, the laws of reflexion and refraction in singly refracting media are uninfluenced by the motion of the earth. The method which I employ will, I hope, be found simpler than Fresnel's; besides it applies easily to the most general case. Fresnel has not given the calculation for reflexion, but has merely stated the result; and with respect to refraction, he has only considered the case in which the course of the light within the refracting medium is in the direction of the earth's motion. This might still leave some doubt on the mind, as to whether the result would be the same in the most general case.

If the æther were at rest, the direction of light would be that of a normal to the surfaces of the waves. When the motion of the æther is considered, it is most convenient to define the direction of light to be that of the line along which the *same portion* of a wave moves relatively to the earth. For this is in all cases the direction which is ultimately observed with a telescope furnished with cross wires. Hence, if A is any point in a wave of light, and if we draw A B normal to the wave, and proportional to V or $\frac{V}{\mu}$, according as the light is passing through vacuum or through a refracting medium, and if we draw B C in the direction of the motion of the æther, and proportional to v or $\frac{v}{\mu^2}$, and join A C, this line will give the direction of the ray. Of course, we might equally have drawn A D equal and parallel to B C and in the opposite direction, when D B would have given the direction of the ray.

Let a plane P be drawn perpendicular to the reflecting or refracting surface and to the waves of incident light, which in this investigation may be supposed plane. Let the velocity v of the æther in vacuum be resolved into p perpendicular to the plane P, and q in that plane; then the resolved parts of the velocity $\frac{v}{\mu^2}$ of the æther within a refracting medium will be $\frac{p}{\mu^2}$, $\frac{q}{\mu^2}$. Let us first consider the effect of the velocity p.

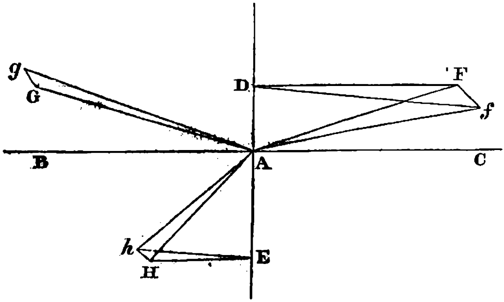
It is easy to see that, as far as regards this resolved part of the velocity of the æther, the directions of the refracted and reflected *waves* will be the same as if the æther were at rest. Let B A C (fig. 1) be the intersection of the refracting surface and the plane P; D A E a normal to the refracting surface; A F, A G, A H normals to the incident, reflected and refracted waves. Hence A F, A G, A H will be in the plane P, and

$$\angle \mathrm{GAD} = \mathrm{FAD}, \quad \mu \sin \mathrm{HAE} = \sin \mathrm{FAD}.$$

Take $$\mathrm{AG} = \mathrm{AF}, \quad \mathrm{AH} = \frac{1}{\mu}\,\mathrm{AF}.$$

Draw Gg, Hh perpendicular to the plane P, and in the direction of the resolved part p of the velocity of the æther, and

Fig. 1.



Ff in the opposite direction; and take

$$Ff : Hh : FA :: p : \frac{p}{\mu^2} : V, \text{ and } Gg = Ff,$$

and join A with f, g and h. Then fA, Ag, Ah will be the directions of the incident, reflected and refracted rays. Draw FD, HE perpendicular to DE, and join fD, hE. Then fDF, hEH will be the inclinations of the planes fAD, hAE to the plane P. Now

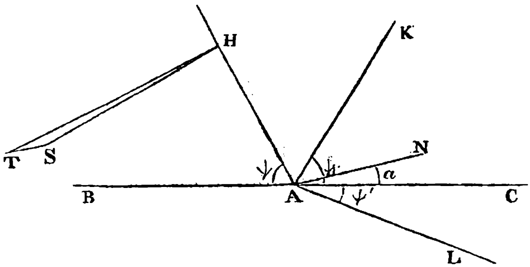
$$\tan FDf = \frac{p}{V \sin FAD}, \quad \tan HEh = \frac{\mu^{-2} p}{\mu^{-1} V \sin HAE},$$

and $\sin FAD = \mu \sin HAE$; therefore $\tan FDf = \tan HEh$, and therefore the refracted ray Ah lies in the plane of incidence fAD. It is easy to see that the same is true of the reflected ray Ag. Also $\angle gAD = fAD$; and the angles fAD, hAE are sensibly equal to FAD, HAE respectively, and we therefore have without sensible error, $\sin fAD = \mu \sin hAE$. Hence the laws of reflexion and refraction are not sensibly affected by the velocity p.

Let us now consider the effect of the velocity q. As far as depends on this velocity, the incident, reflected and refracted rays will all be in the plane P. Let AH, AK, AL be the intersections of the plane P with the incident, reflected and refracted waves. Let ψ, $\psi_{\prime}$, ψ' be the inclinations of these waves to the refracting surface; let NA be the direction of the resolved part q of the velocity of the æther, and let the angle $NAC = \alpha$.

The resolved part of q in a direction perpendicular to AH is $q \sin(\psi + \alpha)$. Hence the wave AH travels with the velocity $V + q \sin(\psi + \alpha)$; and consequently the line of its intersection with the refracting surface travels along A B with the

Fig. 2.



velocity $\operatorname{cosec}\psi\,\{V+q\sin(\psi+\alpha)\}$. Observing that $\frac{q}{\mu^2}$ is the velocity of the æther within the refracting medium, and $\frac{V}{\mu}$ the velocity of propagation of light, we shall find in a similar manner that the lines of intersection of the refracting surface with the reflected and refracted waves travel along A B with velocities

$$\operatorname{cosec}\psi_{\prime}\,\{V+q\sin(\psi_{\prime}-\alpha)\},\quad \operatorname{cosec}\psi'\left\{\frac{V}{\mu}+\frac{q}{\mu^2}\sin(\psi'+\alpha)\right\}$$

But since the incident, reflected and refracted waves intersect the refracting surface in the same line, we must have

$$\left.\begin{aligned}\sin\psi_{\prime}\{V+q\sin(\psi+\alpha)\} &= \sin\psi\{V+q\sin(\psi_{\prime}-\alpha)\},\\ \mu\sin\psi'\{V+q\sin(\psi+\alpha)\} &= \sin\psi\left\{V+\frac{q}{\mu}\sin(\psi'+\alpha)\right\}.\end{aligned}\right\}\quad (A)$$

Draw H S perpendicular to A H, S T parallel to N A, take S T : H S :: q : V, and join H T. Then H T is the direction of the incident ray; and denoting the angles of incidence, reflexion and refraction by ϕ, $\phi_{\prime}$, ϕ', we have

$$\phi-\psi = \text{S H T} = \frac{\text{S T}\sin\text{S}}{\text{S H}} = \frac{1}{V}\times \text{resolved part of } q \text{ along}$$
A H
$$=\frac{q}{V}\cos(\psi+\alpha).\quad \text{Similarly,}$$

$$\phi_{\prime}-\psi_{\prime} = \frac{q}{V}\cos(\psi_{\prime}-\alpha),\quad \phi'-\psi' = \frac{q}{\mu V}\cos(\psi'+\alpha):$$

whence
$$\sin\psi = \sin\phi - \frac{q}{V}\cos\phi\cos(\phi+\alpha),$$

$$\sin\psi_{\prime} = \sin\phi_{\prime} - \frac{q}{V}\cos\phi_{\prime}\cos(\phi_1-\alpha),$$

$$\sin\psi' = \sin\phi' - \frac{q}{\mu V}\cos\phi'\cos(\phi' + \alpha).$$

On substituting these values in equations (A), and observing that in the terms multiplied by q we may put $\phi_{\prime} = \phi$, $\mu \sin \phi' = \sin \phi$, the small terms destroy each other, and we have $\sin \phi_{\prime} = \sin \phi$, $\mu \sin \phi' = \sin \phi$. Hence the laws of reflexion and refraction at the surface of a refracting medium will not be affected by the motion of the æther.

In the preceding investigation it has been supposed that the refraction is out of vacuum into a refracting medium. But the result is the same in the general case of refraction out of one medium into another, and reflexion at the common surface. For all the preceding reasoning applies to this case if we merely substitute $\frac{p}{\mu'^2}$, $\frac{q}{\mu'^2}$ for p, q, $\frac{V}{\mu'}$ for V, and $\frac{\mu}{\mu'}$ for μ, μ' being the refractive index of the first medium. Of course refraction out of a medium into vacuum is included as a particular case.

It follows from the theory just explained, that the light coming from any star will behave in all cases of reflexion and ordinary refraction precisely as it would if the star were situated in the place which it appears to occupy in consequence of aberration, and the earth were at rest. It is, of course, immaterial whether the star is observed with an ordinary telescope, or with a telescope having its tube filled with fluid. It follows also that terrestrial objects are referred to their true places. All these results would follow immediately from the theory of aberration which I proposed in the July number of this Magazine; nor have I been able to obtain any result, admitting of being compared with experiment, which would be different according to which theory we adopted. This affords a curious instance of two totally different theories running parallel to each other in the explanation of phænomena. I do not suppose that many would be disposed to maintain Fresnel's theory, when it is shown that it may be dispensed with, inasmuch as we would not be disposed to believe, without good evidence, that the æther moved quite freely through the solid mass of the earth. Still it would have been satisfactory, if it had been possible, to have put the two theories to the test of some decisive experiment.