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Searching for the Ether: Leopold Courvoiser's Attempts to Measure the Absolute Velocity of the Solar System

The Very Early History of Trigonometry

An Early Use of the Chain Rule

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Searching for the Ether: Leopold Courvoiser's Attempts to Measure the Absolute Velocity of the Solar System

ROBERTO DE ANDRADE MARTINS
Physics Department, State University of Paraiba (UEPB), Brazil
roberto.andrade.martins@gmail.com

Introduction

Leopold Courvoisier (1873-1955) was an observer at the Berlin / Babelsberg astronomical observatory from 1905 up to his retirement in 1938. Most of his work was traditional astrometrical observation resulting in the publication of several star catalogues. A relevant part of his publications was devoted, however, to another subject: the attempt to detect the motion of the solar system through the ether.

Most of Courvoiser's search for measurable effects of the ether was based upon two "principles". According to him, (1) the angles of incidence and reflection of light could be different, relative to the proper reference system of the mirror, if it moved through the ether; and (2) the Lorentz contraction of the Earth due to its motion through the ether produced observable effects relative to the Earth's reference system. Both "principles", of course, violate the principle of relativity. Courvoisier presented theoretical arguments attempting to show that there should exist second order measurable effects. He searched for those effects using both astronomical observations and laboratory experiments and claimed that he had measured a velocity of the solar system of about 600 km/s. This paper presents a description and analysis of Courvoiser's ether researches.

Leopold Courvoisier

Leopold Courvoisier was born on 24 January 1873 in Rihen near Basel (Switzerland).[1] His father Ludwig Georg Courvoisier was a physician and was in charge of the surgery chair of the University of Basel. Leopold (or Leo, as he was usually called) passed away in the same city where he was born, on 31 December 1955. However, most of his professional life was spent in Germany.

Courvoisier exhibited an interest for astronomy since he was 15 years old. In 1891 he began his university studies, first in Basel and later in

[1] For biographical information, see Courvoisier's obituary: Nikolaus Benjamin Richter, "Leopold Courvoisier", *Astronomische Nachrichten*, cclxxxiv (1957), 47-48.

Strasbourg – at a time when this city belonged to Germany. In 1897 he completed his dissertation, on the absolute height of the pole as observed from Strasbourg ("Die absolute Polhöhe von Straßburg"). The next year he became an assistant observer at the Königstuhl astronomical observatory near Heidelberg, under Karl Wilhelm Valentiner. In 1900 he obtained his Doctor degree in Straßburg. From 1905 onward he worked at the Berlin / Babelsberg observatory as an astronomical observer, under the direction of Karl Hermann Struve. In 1913 the Berlin observatory moved to its new site, in Babelsberg,[2] and one year later Courvoiser became its chief observer and professor. He worked at Babelsberg up to his retirement in 1938, when he was 65 years old. In 1943 he moved to his birthplace, where he kept making observations and publishing papers up to his death. Back to Switzerland, he was the editor of several of Leonhard Euler's astronomical works.

Courvoisier's main astronomical contribution was a large series of routine astrometrical observations and the production of star catalogues. Volumes 5, 6 and 7 of Poggendorff's *Biographisch-literarisches Handwörterbuch* provide references of about 10 large works (astronomical catalogues) besides nearly 100 minor contributions by him.[3] However, Courvoisier's work was not restricted to common astrometrical observations. From his tedious measurements there soon came out evidences that he regarded as disproof of the theory of relativity.

Coursoisier did not accept the theory of relativity. He believed there was an ether, and attempted to measure the absolute velocity of the solar system relative to this medium. From 1921 to his death, Courvoisier published a series of over 30 papers where he described the theoretical basis of his search and the several experimental techniques he used in attempting to detect the motion of the Earth relative to the ether. Some of his measurements used astronomical observations; other measurements depended on other physical effects (gravitational, etc.). As a result of his observations he claimed that he had measured a velocity of the solar system of about 600 km/s in a direction close to 75^o right ascension and +40^o declination.

[2] The history of the Berlin / Babelsberg observatory is described in Julius Dick, "The 250th anniversary of the Berlin observatory", *Popular astronomy*, lix (1951), 524-35.

[3] Paul Weinmeister (ed.). *J. C. Poggendorff's biographisch-literarisches Handwörterbuch für Mathematik, Astronomie, Physik, Chemie und verwandte Wissenschaftgebiete (1904 bis 1922)*, vol. 5 (Leipzig, 1926); Hans Stobbe (ed.), *J. C. Poggendorff's biographisch-literarisches Handwörterbuch für Mathematik, Astronomie, Physik mit Geophysik, Chemie, Kristallographie und verwandte Wissenschaftgebiete (1923 bis 1931)*, vol. 6 (Leipzig, 1936-1938); Rudolph Zaunick and Hans Salié (eds.), *J. C. Poggendorff biographisch-literarisches Handwörterbuch der exakten Wissenschaften (1932 bis 1953)*, vol. 7 (Berlin, 1956-1962).

FIG. 1. Leopold Courvoisier (about 30 years old).[4]

The papers describing those researches were published in several scientific journals – especially *Astronomische Nachrichten*, *Physikalische Zeitschrift* and *Zeitschrift für Physik*. His work was largely ignored and had a small impact. A few authors (e.g. Ernest Esclangon and Dayton Miller) who also claimed they had observed effects due to the ether have cited his works.

[4] Portrait painted by Alexander Perandin Moreira, from a photo published in Carl V. Charlier and Folke Engström (eds.), *Porträtgallerie der astronomischen Gesellschaft* (Stockholm, 1904), 17.

Historians of science have also neglected those researches,[5] although they present the largest set of empirical results that was ever published against the theory of relativity by a professional scientist. Courvoisier exhibited an outstanding theoretical and experimental skill, and his results can be regarded as one of the strangest puzzles in the history of relativity.

Courvoisier and relativity

Courvoisier's earliest involvement with relativity was an outcome of his routine measurements of star positions. In the beginning of the twentieth century, Courvoisier had noticed that the right ascension and declination of fixed stars suffered a small influence when they are observed close to the Sun. As this influence had a period of one year, he called it "annual refraction". His first work on the subject was published in 1905,[6] that is, much earlier than the development of the general theory of relativity. In 1911, after the publication of Einstein's early thoughts on the gravitational deflection of light rays, Erwin Freundlich recalled that Courvoisier's work had exhibited an effect that was qualitatively similar to the one predicted by Einstein.[7] Courvoisier interpreted the effect he had measured as due to refraction of light by a denser medium around the Sun, not as a consequence of relativity. It seems that Courvoisier's opposition to Einstein's work grew steadily from this time onward and he became one of the most intransigent supporters of ether theory after the theory of general relativity received strong confirmation (the eclipse measurements), in 1919. Courvoisier's main anti-relativistic work, however, is not directly linked to "annual refraction".[8]
Courvoisier accepted the existence of a static ether, similar to the medium proposed in the early eighteenth century by Augustin Fresnel. That theory led to the conclusion that there could be no first-order influence of the motion through the ether upon optical experiments performed in the Earth. Besides that, the negative outcome of the Michelson-Morley experiment required an additional hypothesis, and Courvoisier accepted that motion

[5] Klaus Hentschel studied some of Courvoisier's works but he did not analyse the researches described in this paper. See Klaus Hentschel, "Freundlich, Erwin, Finlay and testing Einstein's theory of relativity", *Archive for history of exact sciences*, xlvii (1994), 143-201; Klaus Hentschel, *The Einstein tower. An intertexture of dynamic construction, relativity theory, and astronomy* (Stanford, 1997).
[6] Leopold Courvoisier, "Kinemara's Phänomen und die 'jährliche Refraktion' der Fixsterne", *Astronomische Nachrichten*, clxvii (1905), 81-106.
[7] Hentschel, *The Einstein tower* (ref. 5), 10-11.
[8] Klaus Hentschel, *The Einstein tower* (ref. 5), 11, claimed that Courvoisier derived the speed of the Earth's motion through the ether from his data on annual refraction, but his data for the computation of the speed of the Earth was taken from completely independent sources, as will be shown in this paper.

through the ether produced a real contraction of all moving bodies, according to the early explanation proposed by Fitzgerald and Lorentz. According to Lorentz, the principle of relativity would hold exactly for any optical or electromagnetic phenomenon, but Courvoisier did not follow Lorentz's theory in this respect. He directly denied the principle of relativity and attempted to measure the motion of the solar system through the ether using several different techniques.

In 1921 Courvoisier published his first thoughts on the possibility of measuring the absolute velocity of the Earth through the ether.[9] According to Courvoisier's own declaration, his early calculations concerning the motion of the Earth were an outcome of routine work.[10] In 1920 the Leyden Observatory published the details of a large series of observations of stars close to the North Pole that had been made between 1862 and 1874. Those measurements used an old method aiming to reduce observational errors: the stars were observed both with the meridian telescope directly pointed to them, and with the telescope pointed to the images of the stars reflected by a mercury mirror. This double assessment allowed corrections for any changes of the local vertical due to geological motions. It occurred to Courvoisier that those determinations could be used to measure the speed of the Earth through the ether.

Courvoisier assumed that the reflection of light by a mirror could undergo some influence of the motion of the mirror through the ether, *even when the effect was observed relative to the proper reference system of the mirror*. Any observable effect should be of the second order in v/c. It would be impossible to detect such a small effect if the speed of the Earth relative to the ether was about $10^{-4}\,c$ (that is, its orbital velocity), because for usual angle measurements (let us say, 60°) a difference of 10^{-8} would amount to only 0.002" – an effect that could not be observed. However, Courvoisier assumed that there could exist a much larger speed of the whole solar system relative to the ether, and analyzed the data published by the Leyden Observatory searching for some systematic effect.

He computed the difference $z-z'$ between the direct zenith distance z and the reflected zenith distance z' of the stars listed in the catalogue, attempting to find a systematic effect that varied in a periodic way with the sidereal time of observations. Using a graphical method, he did find such an effect, and then he submitted the data to quantitative analysis. He derived an equation to describe the reflection of light in a moving mirror and

[9] Leopold Courvoisier, "Zur Frage der Mitführung des Lichtäthers durch die Erde", *Astronomische Nachrichten*, ccxiii (1921), 281-8; *idem*, "Über astronomische Methoden zur Prüfung der Lichtätherhypothese", *Astronomische Nachrichten*, ccxiv (1921), 33-36.

[10] Leopold Courvoisier, "Ergebnisse von Beobachtungen und Versuchen zur Bestimmung der 'absoluten' Erdbewegung", *Scientia*, xlvii (1930), 165-74; French translation: "Résultats d'observations et d'expériences faites pour la détermination du mouvement 'absolu' de la Terre", *Scientia* (supplément), xlvii (1930), 76-84.

determined the relevant parameters from an analysis of the Leyden data, using the method of minimum squares. He obtained an effect corresponding to a speed of about 800 km/s in the direction of the Auriga constellation. This speed is, of course, much larger than the orbital speed of the Earth. Courvoisier interpreted it as due to the motion of the whole solar system through the ether. A few years later, Courvoisier obtained new data, using the same method (direct versus reflected direction). Using the vertical circle of the Babelsberg observatory, he made a long series of observations (1921-1922) that led to results similar to those that had been obtained from the Leyden observations.

After obtaining his first positive result, Courvoisier attempted to find other independent methods of measuring the speed of the Earth (or the solar system) relative to the ether. He conjectured that the Lorentz contraction of the Earth and of optical instruments could have some small observable influence on astronomical observations. According to Courvoisier, the motion of the Earth relative to the ether produces a contraction that transforms its spherical shape into an ellipsoid with the smaller axis in the direction of its motion. The surface of the ellipsoid, at each point, was supposed to be perpendicular to the local gravitational field. As the Earth rotates, each place on the surface of the Earth passes through different points of the ellipsoid, and the angle between the axis of the Earth and the local vertical direction should undergo a periodical change.

Of course, it is impossible to measure the angle between the local vertical and the axis of rotation of the Earth. However, since the direction of this axis is fairly constant relative to the fixed stars (for short time periods), it is possible to choose a star very close to the North celestial pole and to measure its distance to the zenith (that is, the local vertical direction). This angle, according to Courvoisier's theory, should undergo a periodical change, as a function of the sidereal time.

As a matter of fact, Courvoisier had already measured the position of a star very close to the North pole, in a long series of observations from 1914 to 1917, using the Babelsberg Observatory vertical circle.[11] Those measurements were very accurate and were evenly distributed as regards the sidereal time of the observations. They were therefore suitable for looking for the influence of the Lorentz contraction on astronomical measurements.

As in the former case, Courvoisier first plotted the zenithal distances of the star against sidereal time, and found a regular fluctuation of the angle. He

[11] Leopold Courvoisier, "Zenitdistanzbeobachtungen der Polarissima am Vertikalkreise der Sternwarte Berlin-Babelsberg", *Astronomische Nachrichten*, ccviii (1919), 349-64. He made this series of measurements as routine observations to ascertain the latitude of the Babelsberg observatory. The method used by Courvoisier is very precise, and was recently used for the determination of the azimuth of a transit instrument in Brazil: Ramachrisna Teixeira and Paulo Benevides Soares, "Absolute azimuth determination", *Astronomy and astrophysics*, clxv (1986), 251-3.

then developed an equation to account for the effect, analyzed the data using the minimum square method, and obtained his second measurement of the velocity of the Earth relative to the ether. The speed obtained in this case was about 700 km/s, in the direction of the constellation of Perseus (not very far from Auriga). Courvoisier regarded the agreement of those two earliest results as satisfactory, and this led him to further researches. There was a delay of 5 years between Courvoisier's first positive results and his next publication on the subject.[12] In this period he accumulated a series of positive results by different methods, obtained the equations required for the analysis of his data, and devised new methods for measuring the absolute speed of the Earth. This delay shows that Courvoisier was careful enough to resist publishing preliminary results before he was able to amass a large amount of evidence for his claim.

The method of the moving mirror

Courvoisier derived equations[13] that related the relevant measurements to the parameters of the motion of the Earth relative to the ether.[14] The main parameters that appear in his equations (Fig. 2) are:

c = the speed of light relative to the ether = 300,000 km/s
v = speed of the Earth (or the solar system) relative to the ether
A = right ascension of the apex of the absolute motion
D = declination of the apex of the absolute motion
α = North local component of v/c
β = Zenith local component of v/c
γ = West local component of v/c
ϕ = latitude of the terrestrial observatory
θ = sidereal time of measurement

A straightforward geometrical analysis shows that the components of v/c are:

$$\alpha = (v/c)\,[\cos\phi \sin D - \sin\phi \cos D \cos(\theta - A)] \qquad (1)$$

$$\beta = (v/c)\,[\sin\phi \sin D + \cos\phi \cos D \cos(\theta - A)] \qquad (2)$$

[12] Leopold Courvoisier, "Bestimmungsversuche der Erdbewegung relativ zum Lichtäther", *Astronomische Nachrichten*, ccxxvi (1926), 241-64.
[13] Courvoisier never published the details of his derivations – he only presented his main assumptions, a few steps and the final results. In all relevant cases, however, I have been able to confirm that his equations do follow from his assumptions.
[14] Courvoisier, "Bestimmungsversuche der Erdbewegung relativ zum Lichtäther" (ref. 12).

$$\gamma = -(v/c)\cos D \sin(\theta - A) \qquad (3)$$

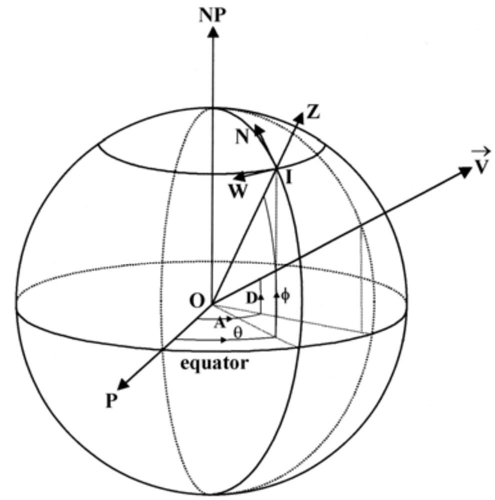


FIG. 2. This diagram shows the main geometrical parameters used in Courvoisier's theoretical analysis of ether effects. The spherical surface represents the Earth, and the observer is at *I*, and the local directions *Z*, *N*, *W* correspond to Zenith, geographical North and West. The North Pole is in the direction *NP*. The velocity of the Earth is $\vec{V}$.

In Courvoisier's first method, as described above, light was reflected by a mirror. To derive the theoretical effect, it was necessary to study the influence of the motion of the mirror through the ether upon the direction of the reflected ray. Courvoisier made use of the non-relativistic analysis developed by Adolf von Harnack,[15] that predicted that the angle of reflection would be different from the angle of incidence, relative to the proper reference system of the mirror (Fig. 3). This was one of Courvoisier's main assumptions that was incompatible with the principle of relativity.

[15] Adolf von Harnack, "Zur Theorie des bewegten Spiegels", *Annalen der Physik*, series 4, xxxix (1912), 1053-8.

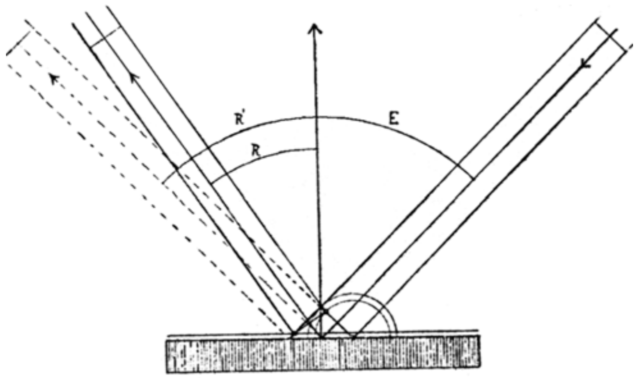


FIG. 3. Following a theoretical analysis by Adolf von Harnack, Courvoisier accepted that the angle of reflection of light in a moving mirror is influenced by its motion through the ether, and that there is a second-order effect that can be measured in the reference frame of the mirror.

Taking into account this "principle of the moving mirror", Courvoisier predicted that the angle between the local vertical (zenith) and the direction of observation of a given star would be slightly different from the angle between the zenith and the direction of the star observed using a mercury mirror (Fig. 4).

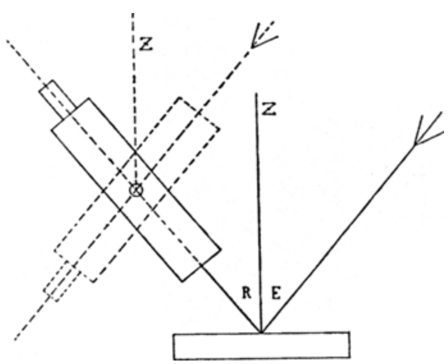


FIG. 4. Courvoiser compared the direct measurement of the direction of a star with its direction observed by reflection on a mercury mirror.

In this specific case, the contraction of the Earth could produce no effect, because both measurements were made relative to the same reference (the local vertical) and the surface of the mercury mirror is, of course, perpendicular to the local vertical, whatever the changes that the gravitational field could undergo due to Lorentz contraction. The predicted effect was a small systematic difference between the direct and the reflected angles, which should depend on the direction of the observatory relative to the motion of the Earth through the ether.

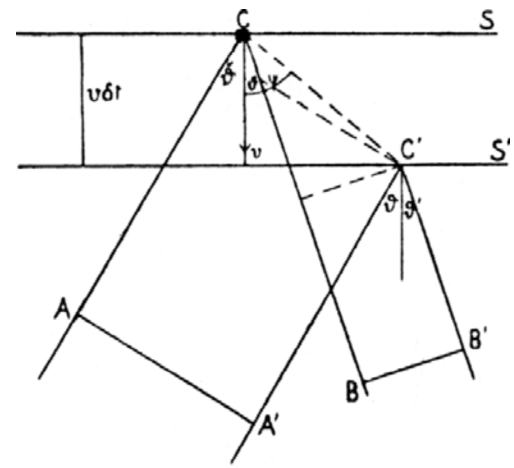


FIG. 5. Harnack's diagram for analyzing the reflection of light in a moving mirror. The initial position of the mirror is *S*, and after a time δt its position is *S*'. *AA*' is a wave front of the incident light beam, and *BB*' is a wave front of the reflected beam.

Let θ be the angle of incidence and θ' the angle of reflection of a light ray in a moving mirror, measured relative to the ether (Fig. 5).[16] According to Harnack's analysis, instead of θ=θ' the following equations would hold:

$$\sin\theta' = (1 - \beta^2)\sin\theta \,/\, (1 + 2\beta\cos\theta + \beta^2) \qquad (4)$$

$$\cos\theta' = [(1 + \beta^2)\cos\theta + 2\beta] \,/\, (1 + 2\beta\cos\theta + \beta^2) \qquad (5)$$

[16] In his equations Courvoisier used θ as a symbol of sidereal time, but in this particular derivation we are following Harnack's notation in his paper "Zur Theorie des bewegten Spiegels" (ref. 15).

In those equations, the speed of the mirror is $\beta = v/c$, in the direction perpendicular to the mirror. Any motion of the mirror parallel to its surface would have no influence upon the direction of light. In the case of the mercury mirror, the relevant direction if the local vertical, and therefore β, here, has the same general meaning ascribed by Courvoisier to this symbol. Relative to the proper reference system of the mirror there is an aberration effect, and the angles of incidence (z) and reflection (z') are:

$$z = \theta + \alpha \cos \theta - \beta \sin \theta \qquad (6)$$

$$z' = \theta' + \alpha \cos \theta' + \beta \sin \theta' \qquad (7)$$

where α is component of the velocity v/c of the mirror parallel to its surface. Notice that this is the classical aberration effect. A relativistic analysis would lead to a different result.

The measured effect is the difference between z' and z:

$$z' - z = (\theta' - \theta) + \alpha (\cos \theta' - \cos \theta) + \beta (\sin \theta' - \sin \theta) \qquad (8)$$

Taking into account the above equations and making suitable substitutions, one obtains the approximate result:

$$z' - z = 2\alpha\beta \sin^2 z \qquad (9)$$

Replacing α and β by their values in Eqs. (1) and (2),[17] one obtains:

$$z' - z = [(v/c)^2 \sin^2 z].[\sin 2\phi.\sin^2 D + \cos 2\phi.\sin 2D.\cos (\theta - A) -$$

$$- \sin 2\phi.\cos^2 D.\cos^2(\theta - A)] \qquad (10)$$

Notice that this equation contains a constant term and two periodical components with different periods – one sidereal day [cos (θ–*A*)] and half a sidereal day [cos² (θ–*A*)]. Therefore, from a suitable analysis of the data it should be possible to obtain the speed (v/c), the declination (D) and the right ascension (A) of the motion of the Earth relative to the ether.

Repetition of the Leyden measurements

The Leyden measurements had used four stars close to the North Pole. The difference $z - z'$ was measured in a series of observations, at the times of upper and lower culmination of each star. The observed values of the periodical components of $z - z'$ amounted to less than 1", varying from 0.04" for one of the stars to about 0.5" for another. The error of the measurements was estimated as 0.01", therefore the effect was regarded as significant. From the Leyden data Courvoisier obtained the results:

$$A = 104° \pm 21°; D = +39° \pm 27°; v = 810 \pm 215 \text{ km/s}$$

[17] From this point onward, θ is used again to represent sidereal time.

The estimated error of the speed amounted to about 25%. The errors of the right ascension and declination amounted to about 1/15 of the full circle.

Between 1921 and 1922 Courvoisier repeated the Leyden measurements, but with a slight change of method. Instead of a meridian circle he used a Wanschaff vertical circle that enabled him to make measurements of the stars at any time during the night. Therefore his measurements were not limited to two sidereal times for each star.

From 4 June to 14 December 1921 he made a series of 142 measurements of the polar star BD +89.3°, and from 18 March to 23 May 1922 he made further 64 determinations of $z - z'$. From those measurements Courvoisier obtained:

$$A = 93° \pm 7°; D = +27° \pm 12°; v = 652 \pm 71 \text{ km/s}$$

The estimated relative error of the speed was reduced to about 10% and the errors of the right ascension and declination amounted to less than 1/30 of the full circle.

Courvoisier's work called the attention of a French astronomer, the director of the Strasbourg observatory, Ernest Esclangon, who repeated those measurements.[18] He confirmed the existence of a systematic effect of the same order of magnitude, and computed the values of A=69° and D=44°. Esclangon did not publish the estimated errors of his evaluation, nor the estimated speed of the Earth.

Other evaluations were later obtained by Courvoisier using measurements made at München (1930-1931) and Breslau (1933-1935), with the following results:

München	Breslau (1)	Breslau (2)
A = 73° ± 6° D = +40° (estimated)[19] v = 889 ± 93 km/s	A = 92° ± 12° D = +44° ± 25° v = 927 ± 200 km/s	A = 80° ± 4° D = +30° ± 10° v = 700 ± 60 km/s

The results obtained in the second Breslau series presented the smallest errors.

In 1945, after his retirement, Courvoisier made a final series of observations from Basel. He obtained the following results:

$$A = 60° \pm 14°; D = +40° \text{ (estimated)}; v = 656 \pm 157 \text{ km/s}$$

[18] Ernest Esclangon, "Sur la dyssimétrie mécanique et optique de l'espace en rapport avec le mouvement absolu de la Terre", *Comptes rendus de l'academie des sciences de Paris*, clxxxii (1926), 921-3.

[19] In some of his analysis, Courvoisier found that the effect with one sidereal day period was not clearly noticeable. In those cases, he assumed the value of 40° for the declination, and computed the right ascension and speed of the Earth.

If we compare all the series of measurements, we notice that the right ascension varied between 60° and 104° (more than the estimated errors); the declination varied between 39° and 44° (within the estimated errors);[20] and the speed varied between 652 and 927 km/s (within estimated errors). Notice that it is very hard to explain away Courvoisier's results as due to instrument errors, because the observed effect varied with periods of one sidereal day and half sidereal day. All common causes of error (gravity changes, temperature changes, etc.) would vary with periods of one (or half) solar day. Tidal influences due to the Moon would have periods that could also be easily distinguished from the effects predicted by Courvoisier. Besides that, the data used by Courvoisier was obtained with different instruments at different places, and covered a time span of 80 years. The results presented by Courvoisier are therefore highly impressive and cannot be dismissed lightly.

Courvoisier's device for measuring the absolute speed of the earth

In the first method used by Courvoisier, the stars work as mere point-like light sources. There is nothing peculiarly “astronomical” in the observed effect because, according to Courvoisier's theory, this was ascribed to the “principle of the moving mirror”. Therefore, similar effects should occur for terrestrial light sources, too.
Accordingly, Courvoisier was led to build a new instrument: an optical device for measuring absolute motion (Fig. 6).[21] He used two small telescopes that were placed in an underground room where the temperature was fairly constant. Both telescopes pointed obliquely (zenithal distance = 60°) to a mercury mirror that was placed between them. They were mounted in a vertical plane in the East-West direction. One of the telescopes had a small electric light close to its reticule, and this was the light source that was observed from the second telescope. Both telescopes were first adjusted so that it was possible to see the reflection of the illuminated reticule of the first telescope from the second telescope. They were then fastened in those directions. Of course, the angles of the telescopes with the local vertical were sensibly equal. The experiment did not try to measure any difference between those angles. It attempted to detect small periodical changes of the position of the image of the first telescope reticule as observed from the second one. The apparent motion of

[20] The slight variations of the values found for the declination led Courvoisier to assume this value as known, as remarked above (note 18), in all cases when it was impossible to compute A, D and v/c.
[21] Leopold Courvoisier, “Bestimmungsversuche der Erdbewegung relativ zum Lichtäther II”, *Astronomische Nachrichten*, ccxxx (1927), 425-32; *idem*, “Über die Translationsbewegung der Erde im Lichtäther”, *Physikalische Zeitschrift*, xxviii (1927), 674-80.

the reticule was measured with the aid of the ocular micrometer of the second telescope.
Using this device, Courvoisier made two series of observations in 1926 and 1927. Afterwards, he had a special instrument built for this purpose, and made a third series of observations in 1932.
In his first experiments the telescopes were placed in a vertical plane in the East-West direction. In 1926 and 1928 Courvoisier built two new instruments that could be rotated. He expected that this would improve his measurements. However, he found out that it was impossible to compare measurements when the device was rotated, due to mechanical problems, and the instruments could only be effectively used in a fixed position.
The equation used to compute the effect was similar to that used in the case of the observation of stars, but instead of the North component of the speed, it was necessary to take into account the West component. As in the former case, the resulting equation has a constant term plus variable components with periods of one sidereal day and half sidereal day.

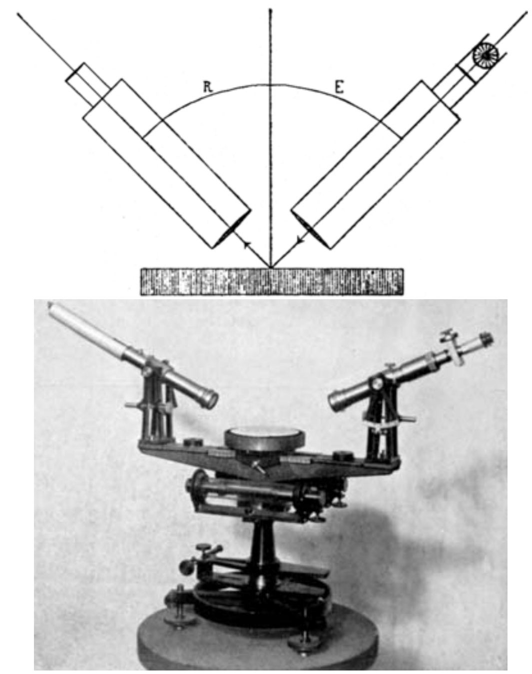


FIG. 6. Courvoisier’s double telescope apparatus for measuring the motion of the Earth through the ether.

TABLE 1. Measurements made by Courvoisier in 1926 with the double telescope instrument.

First series:		
Sidereal time θ	**(z – z') + constant**	**number of measurements**
0.32 h	– 0.08”	21
1.23 h	+ 0.04”	64
2.45 h	+ 0.07”	14
3.31 h	– 0.38”	56
4.28 h	– 038”	14
5.28 h	– 0.57”	68
7.37 h	– 0.58”	55
9.29 h	– 0.57”	64
11.24 h	– 0.24”	30
12.73 h	– 0.04”	20
21.91 h	+ 0.21”	38
23.32 h	+ 0.08”	45

TABLE 2. Measurements made by Courvoisier in 1927 with the double telescope instrument.

Second series:		
Sidereal time θ	**(z – z') + constant**	**number of measurements**
2.9 h	+ 1.54”	4
7.3 h	+ 0.28”	6
8.2 h	+ 0.28”	7
9.1 h	– 0.01”	7
10.1 h	+ 0.23”	6
11.4 h	+ 0.56”	5
12.3 h	+ 0.60”	5
13.7 h	+ 0.52”	7
15.5 h	+ 0.84”	6
17.9 h	+ 0.88”	7
19.9 h	+ 0.80”	7

The first series of measurements was made from 31 July and 6 August 1926, with observations spanning between 3 and 20 o'clock sidereal time; the second one, from 28 February to 29 May 1927, with observations covering the period from 21 to 13 o'clock sidereal time. Both series comprised more than 500 measurements. Tables 1 and 2 shows the mean results obtained by Courvoisier for each sidereal time:

The first series comprised 489 observations, and the second series only 67 observations. From the first series, Courvoisier computed the following values:

$$A = 70° \pm 6°;\ D = +33° \pm 11°;\ v = 493 \pm 54 \text{ km/s}$$

From the second series, he obtained the results:

$$A = 22° \pm 6°;\ D = +72° \pm 11°;\ v = 606 \pm 45 \text{ km/s}$$

Of course, the results obtained from the first series of measurements seemed more reliable than those from the second series, and they exhibited a closer agreement with former measurements.

Notice that, although those measurements attempted to detect the same kind of effects as the astronomical observations – that is, a difference between angle of incidence and angle of reflection in a moving mirror – the star observations used the North-South direction, and the cave experiments employed the East-West direction. The equations were different, and nevertheless Courvoisier obtained a nice agreement between the new device and the former results.

The double mirror experiments

In 1928 Courvoisier built another device to measure the speed of the Earth using the principle of the moving mirror. Instead of using two telescopes, he used a single telescope, with two perpendicular mirrors in front of its objective (Fig. 7).[22] The body of the telescope was placed in a horizontal position. The mirrors were adjusted so that it was possible to observe the reflected image of the thread micrometer of the telescope in close coincidence with the real micrometer thread. He predicted that the relative position of the image and the thread should undergo periodic fluctuations, and computed the predicted effect.

From April to June 1928 Courvoisier obtained a series of 53 measurements, both in the North-South and in the East-West directions, and he computed the following values:

$$A = 74° \pm 1°;\ D = +36° \pm 1°;\ v = 496 \pm 10 \text{ km/s}$$

[22] Leopold Courvoisier, “Bestimmungsversuche der Erdbewegung relativ zum Lichtäther III”, *Astronomische Nachrichten*, ccxxxiv (1928), 137-44.

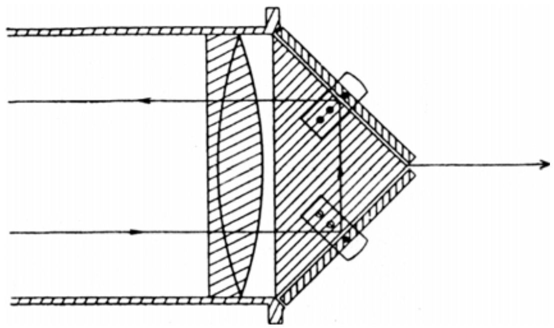


FIG. 7. Courvoisier's coupled mirror device for measuring the motion of the Earth through the ether.

Courvoisier's new experiment was probably suggested by a similar arrangement that had been used by Esclangon in 1927.[23] The French astronomer used two mirrors, but light underwent three reflections (Fig. 8). The maximum effect occurred at 3 h or 15 h sidereal time, corresponding to $A = 45^{o}$ or 225°. Esclangon did not compute the speed of the Earth through the ether – indeed, he did not even provide a definite interpretation of the phenomenon.

The second method: Lorentz contraction

As described above, Courvoisier's second attempt to measure the absolute velocity of the Earth was grounded upon his analysis of the Lorentz contraction of the Earth (Fig. 9). In this case, Courvoisier supposed that the local vertical would undergo a change, due to the Lorentz contraction of the Earth, and this change would be observable as a periodical fluctuation in the angle between the North Pole and the zenith, as a function of the sidereal time.

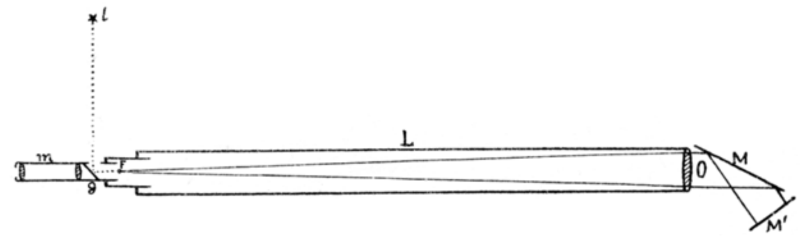
Courvoisier's theoretical analysis led him to predict that the variation of the zenithal distance Δz of a star close to the North Pole would obey the approximate relation:

$$\Delta z = \tfrac{1}{2}\alpha\beta \qquad (11)$$

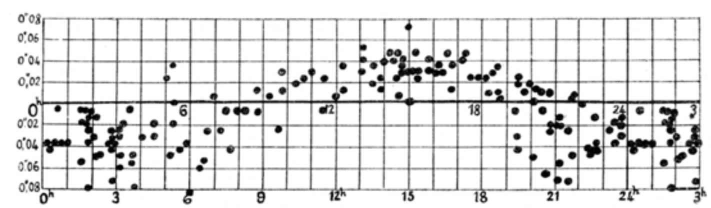
[23] Ernest Esclangon, "Sur la dissymétrie optique de l'espace et les lois de la réflexion", *Comptes rendus de l'académie des sciences de Paris*, clxxxv (1927), 1593-5 ; *idem*, "Sur l'existence d'une dissymétrie optique de l'espace", *Journal des observateurs*, xi (1928), 49-63.

There are some special observational difficulties in this second method. If it were possible to observe a star laying *exactly* in the direction of the celestial North Pole, the observation would be quite simple. However, if the star is not exactly in the direction of the pole, its zenithal distance will depend on the sidereal time of the observation. This classical large effect would have, therefore, a period of one sidereal day and would interfere with any attempt to measure any influence due to the motion through the ether with a period of one sidereal day. Other interfering effects, such as temperature changes, vary with a period of about one solar day, and they are very large and irregular. For those reasons, Courvoisier gave up the attempt of finding the amplitude of the sidereal day effect, and only computed the half sidereal day effect. It was impossible, therefore, to find all parameters, and he assumed a value of 40° for the declination, and computed the speed and right ascension of the motion of the Earth relative to the ether. Dropping out the component corresponding to the period of one sidereal day, he obtained the following equation:

$$\Delta z = -(1/4)(v/c)^2.\sin 2\phi\,(\text{const.} - \cos^2 D.\cos^2(\theta - A)] \qquad (12)$$



a)



(b)

FIG. 8. Esclangon's coupled mirror device for measuring the motion of the Earth through the ether (a), and a graphical representation of his results (b), showing the observed angular fluctuations as a function of sidereal time.

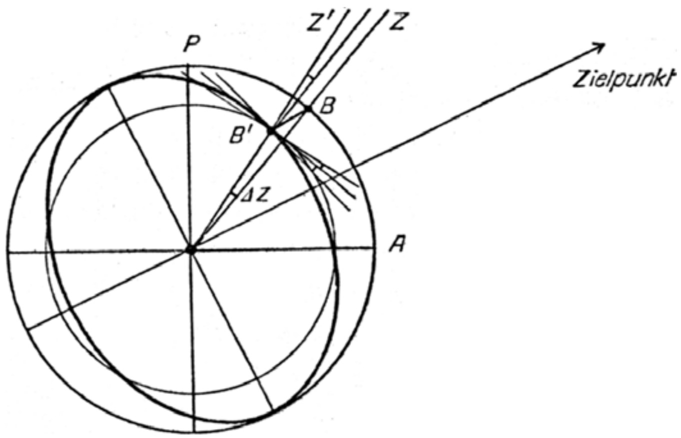


FIG. 9. According to Courvoisier, the Lorentz contraction of the Earth and of optical instruments could have a small observable influence on astronomical observations and terrestrial experiments.

Using the data he had already obtained from 1914 to 1917, and combining those results with other measurements he made in 1921-1922 and 1925-1926, with the same instrument, Courvoisier obtained the following result:

$$A = 74° \pm 3°;\ [D = +40°];\ v = 587 \pm 48 \text{ km/s}$$

He also analyzed measurements that had been obtained in routine observations at the Paris observatory, in the period 1899-1901. All those series of observations exhibited similar variations with a period of 12 sidereal hours. Assuming a value of 40° for the declination, he obtained the following results:

$$A = 70° \pm 11°;\ [D = +40°];\ v = 810 \pm 166 \text{ km/s}$$

Afterwards Courvoisier also computed the motion of the Earth using measurements from Breslau (1923-1925 and 1933-1935) and from München (1927-1931). Taking into account all the observations, he obtained the following final result:

$$A = 65° \pm 10°;\ [D = +40°];\ v = 574 \pm 97 \text{ km/s}$$

Comparison between measurements from different places

The effects predicted by Courvoisier as a consequence of the Lorentz contraction of the Earth should depend on the latitude of the observatory. For that reason, if the same set of stars was observed from two observatories at very different latitudes, there should exist a systematic difference between the measured declinations of the stars, as a function of sidereal time.

To test the existence of this effect, Courvoisier analyzed the catalogues containing measurements made at Heidelberg ($\phi_1 = +49.24°$) and at Cape Town, South Africa ($\phi_2 = -33.48°$). Let D_1 be the declination of some star measured from Heidelberg, and D_2 the declination of the same star measured from Cape of Good Hope. Each declination, according to Courvoisier's analysis, undergoes a periodical change:

$$\Delta z_1 = ½\, \alpha_1\beta_1 \qquad \Delta z_2 = ½\, \alpha_2\beta_2 \qquad (13)$$

Those effects are not equal; therefore, the difference between the declinations measured at the two observatories should undergo a periodical change:

$$D_1 - D_2 = ½\,(\alpha_1\beta_1 - \alpha_2\beta_2) \qquad (14)$$

Using the typical values A=75° and D=40° obtained in former measurements, and taking into account the latitudes of Heidelberg and Cape Town, Courvoisier predicted that there should exist a difference between the measured declinations of the stars that should depend on their right ascension α:

$$D_1 - D_2 = +0.16'' - 0.18''.\cos(\alpha - 5\,h) - 0.16''.\cos 2(\alpha - 5\,h) \qquad (15)$$

The amplitude was obtained by comparing the astronomical data of the two observatories, and led to v =750 km/s. Table 3 contains Courvoisier's comparison between the observed and predicted values of D_1–D_2.

The third column of the table presented the observed values corrected for null declination, in order to avoid classical errors due to atmospheric refraction, etc. There is a better agreement between the theoretical prediction and the corrected values than with the raw data.

Nadir observations

In his analysis of the second method, Courvoisier assumed that the Lorentz contraction of the Earth produces a local periodical change of the direction of the gravitational field. This effect was not compensated by changes in the direction of the astronomical instruments. Therefore, he was led to think that the effect could also be detected in an experiment using a terrestrial light source.

He placed a mercury mirror directly below the observatory meridian circle and pointed the telescope downward. The instrument was then delicately adjusted in such a way that it was possible to observe the reflected image of the micrometer threads superimposed to the real threads. The position of the telescope was locked, and observations were made of the relative displacement of the micrometer thread and its image. He predicted the following deflection in the East-West direction:

$$\Delta z = -(1/4)(v/c)^2.[\sin\phi.\sin 2D.\sin(\theta - A) + \cos\phi.\cos^2 D.\sin 2(\theta - A)] \quad (16)$$

TABLE 3. Difference between the declinations of a star (D_1–D_2), observed from two distant observatories, as a function of sidereal time α.

α	D_1–D_2		
	observed	**observed (corrected)**	**prediction**
0 h	+ 0.35”	+ 0.35”	+ 0.26”
1 h	+ 0.21”	+ 0.21”	+ 0.16”
2 h	+ 0.01”	+ 0.01”	+ 0.04”
3 h	– 0.07”	– 0.07”	– 0.07”
4 h	– 0.17”	–0.17”	–0.16”
5 h	+ 0.03”	+ 0.03	– 0.17”
6 h	+ 0.17”	+ 0.17	– 0.14”
7 h	– 0.03”	– 0.03”	– 0.06”
8 h	+ 0.07”	+ 0.07”	+ 0.04”
9 h	+ 0.10”	+ 0.10”	+ 0.14”
10 h	+ 0.08”	+ 0.08”	+ 0.25”
11 h	+ 0.09”	+ 0.09”	+ 0.32”
12 h	+ 0.29”	+ 0.29”	+ 0.34”
13 h	+ 0.32”	+ 0.35”	+ 0.32”
14 h	+ 0.29”	+ 0.39”	+ 0.29”
15 h	– 0.04”	+ 0.22”	+ 0.25”
16 h	– 0.21”	+ 0.13”	+ 0.20”
17 h	– 0.23”	+ 0.18”	+ 0.19”
18 h	– 0.29”	+ 0.12”	+ 0.20”
19 h	– 0.31”	+ 0.10”	+ 0.23”
20 h	– 0.17”	+ 0.17”	+ 0.29”
21 h	+ 0.04”	+ 0.30”	+ 0.33”
22 h	+ 0.26”	+ 0.36”	+ 0.34”
23 h	+ 0.38”	+ 0.41”	+ 0.32”

Courvoisier made two series of observations: 22-24 October and 22-25 November 1922. He noticed that temperature changes affected the position of the telescope, and that this influence had to be taken into account. From the uncorrected observed measurements he computed the following values:

$$A = 74° \pm 10°;\ D = +67° \pm 13°;\ v = 920 \pm 73 \text{ km/s}$$

Applying a temperature correction, he obtained the following results:

$$A = 98° \pm 7°;\ D = +25° \pm 11°;\ v = 500 \pm 47 \text{ km/s}$$

This experiment was repeated by August Kopff, of the Heidelberg observatory, from 10 to 29 June 1923. As in the case of Courvoisier's experiment, there was a strong effect due to temperature changes (temperature varied between +6°C and +17°C). Courvoisier analyzed Kopff's data assuming the values A = 75° and D = +40°. After applying temperature corrections, he obtained a speed of 753 ± 57 km/s.

Other methods

Courvoisier also attempted to detect the motion of the Earth relative to the ether by other methods. He regarded the positive result of the nadir observation method as a confirmation of his hypothesis that the Lorentz’s contraction produced an observable periodical change of the local vertical. He soon devised other ways of observing such an effect.

Plumb line motion

One of the instruments he used was a plumb line attached to one of the columns of the Babelsberg observatory. The main body of the plumb line was a metallic rod, 95 cm long. At its lower end there was a mark that was illuminated and projected upon a wall. It was possible to observe deflections of about 0.05" of the direction of the plumb line, in the East-West direction.[24] Measurements made in 1925 with this instrument led to a speed of the Earth of about 400 km/s, assuming A = 75° and D = +40°. In 1931 Courvoisier improved this instrument observing the motion of its tip with the aid of a microscope (Fig. 10). Now he was able to compute the three parameters of the Earth's motion, obtaining:

$$A = 64° \pm 6°;\ D = +50° \pm 9°;\ v = 367 \pm 29 \text{ km/s}$$

[24] Leopold Courvoisier, “Ableitung der Bahngeschwindigkeit der Erde aus der auf Grund der Lorentz-Kontraktion (Zeigerstabversuch) betimmten Absolutbewegung”, *Astronomische Nachrichten*, ccxlvii (1932), 105-18.

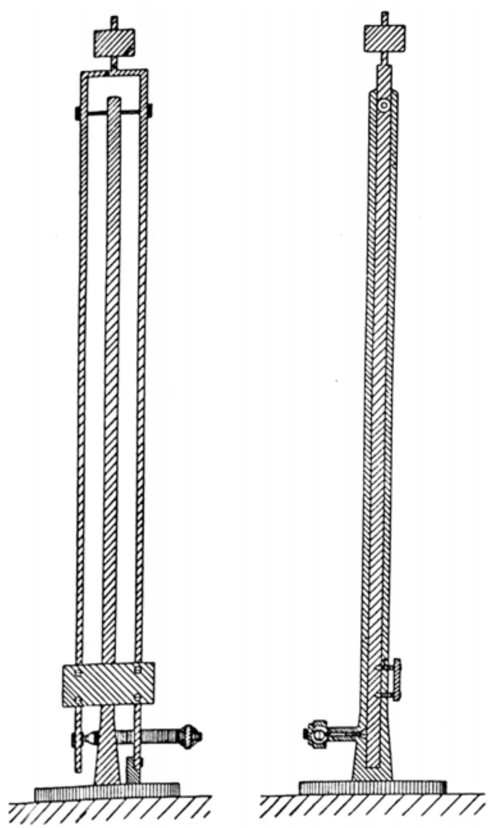


FIG. 10. Courvoisier's plumb line apparatus for measuring oscillations of the local gravitational vertical due to Lorentz contraction.

Similar observations were made by Esclangon, with the help of André-Louis Danjon, using two horizontal pendulums with perpendicular motions.[25] One of the pendulums lead to *A*=69°; for the second pendulum, *A*=52°. Esclangon did not provide other information and did not attempt to compute the speed of the Earth.

[25] Ernest Esclangon, "Sur la dyssimétrie mécanique et optique de l'espace en rapport avec le mouvement absolu de la Terre", *Comptes rendus de l'académie des sciences de Paris*, clxxxii (1926), 921-3.

Bubble level

Another way of observing the variation of the local vertical direction, according to Courvoisier, was with the aid of bubble levels.[26] He used two very sensitive level meters. One of them was attached to the floor of the Babelsberg underground clock room, and the other one was attached in a horizontal position to one of the columns of the same room. Courvoisier measured the difference between the marks of the two level meters. The maximum predicted effect was about 0.30", and with the delicate instruments used by Courvoisier it was possible to measure angular changes as small as 0,03". In the first series of measurements between 15 and 26 June 1929, Courvoisier obtained the following results:

$$A = 59° \pm 6°;\ D = +51° \pm 9°;\ v = 446 \pm 34 \text{ km/s}$$

Comparison between pendulum clocks at different places

According to Courvoisier's hypothesis, the Earth undergoes a real contraction in the direction of its motion through the ether, and this contraction would produce observable periodical changes of the local value of gravity as a function of sidereal time. Pendulum clocks at different places of the Earth should show slightly different readings, and their phases should exhibit a periodical relative fluctuation. Courvoisier analyzed data on pendulum clocks of different astronomical observatories, in an attempt to detect this effect.

Using radio signals it was possible to compare the rates of clocks at very distant observatories. The Annapolis Observatory emitted regular time signals from its pendulum clocks. It was possible to compare the rate of those pendulums to those at another place. Courvoisier asked the help of Bernhard Wanach, from Potsdam, who compared the rate of the pendulum clocks of that observatory to the signals received from Annapolis, from September 1921 to November 1922.[27] Courvoisier's analysis of Wanach's data led to the following results:

$$A = 56° \pm 12°;\ D = +40° \text{ (estimated)};\ v = 873 \pm 228 \text{ km/s}$$

Afterwards, a comparison was made using a comparison between the clocks of Annapolis, Potsdam, Ottawa, and Bordeaux. The mean result obtained by Courvoisier was:

$$A = 81° \pm 5°;\ D = +34° \pm 5°;\ v = 650 \pm 50 \text{ km/s}$$

[26] Leopold Courvoisier, "Bestimmungsversuche der Erdbewegung relativ zum Lichtäther IV", *Astronomische Nachrichten*, ccxxxvii (1930), 337-52; *idem*, "Ist die Lorentz-Kontraktion von Brehungsindex abhängig?", *Zeitschrift für Physik*, xc (1934), 48-62.

[27] Leopold Courvoisier, "Bestimmungsversuche der Erdbewegung relativ zum Lichtäther II", *Astronomische Nachrichten*, ccxxx (1927), 425-32.

Much later, Courvoisier presented another confirmation of this effect. He compared the catalogues of time correction of the observatories of Greenwich, Potsdam, Buenos Aires and Mount Stromslo for the period from 1948 to 1954.[28] There was a nice agreement between the theoretical predictions and the observed time differences, especially in the case of the years 1951-1954.

Local comparison between pendulum clock and chronometer
Courvoisier supposed that the rate of pendulum clocks would vary because of the periodical gravity changes, but mechanical chronometers should not suffer similar changes. Therefore it should be possible to observe effects due to the absolute motion of the Earth comparing pendulum clocks to mechanical chronometers at a single place. Comparisons were made both at Babelsberg and at Potsdam (with the help of Wanach). In his analysis, Courvoisier assumed the value $D = +40°$ and obtained $A = 104° \pm 9°$ and $v = 750$ km/s.

Gravimetric observations
If the Lorentz contraction of the Earth produces gravitational effects, then it should be possible to find its influence on the tides. Esclangon analyzed a set of 166,500 tide measurements, made at Pola, on the Adriatic sea, from 1898 to 1916. He obtained a term with the period of on sidereal day, that could not be associated with the Sun or the Moon, and ascribed it to a "dissymmetry of space".[29] This tidal effect could be described as:

$$48 \text{ mm.}\cos (t - 146.1^{o}) + 25 \text{ mm.}\cos (t - 244.6^{o}) \quad (16)$$

If the local gravity undergoes periodic changes, it should be possible to detect this effect with sensitive gravimeters. In 1927 Courvoisier (with the help of Sergei Gaposchkin) attempted for the first time to measure gravity variations using a very sensitive torsion gravimeter.[30] The instrument could detect a change $\Delta g/g$ of $3\text{x}10^{-6}$, corresponding to a displacement of 0.2 mm of the gravimeter pointer. From a series of measurements undertaken from 1927 to 1928 Courvoisier computed the following values:

$$A = 62° \pm 5°;\ D = +32° \pm 8°;\ v = 543 \pm 55 \text{ km/s}$$

[28] Leopold Courvoisier, "Der Einfluss der 'Lorentz-Kontraktion' der Erde auf den Gang der Quarzuhren", *Experientia*, ix (1953), 286-7; xiii (1957), 234-5.
[29] Ernest Esclangon, "La dissymétrie de l'espace sidéral et le phénomène des marées", *Comptes rendus de l'académie des sciences de Paris*, clxxxiii (1926), 116-18.
[30] Leopold Courvoisier, "Über die Translationsbewegung der Erde im Lichtäther", *Physikalische Zeitschrift*, xxviii (1927), 674-80.

In 1932 Courvoisier obtained new results, taking into account in this new paper some effects due to temperature and humidity. The new results obtained by him were

$$A = 50° \pm 7°;\ D = +45° \pm 18°;\ v = 498 \pm 78 \text{ km/s}$$

For the first time, Courvoisier's results were criticized and checked. In 1932, Rudolf Tomaschek and Walter Schaffernicht reported gravity measurements made with a new kind of gravimeter that was able to detect changes $\Delta g/g$ of 10^{-8}. The instrument was placed inside a cave in a mountain, where the temperature was constant to 0.001° C. No effect of the order of magnitude predicted by Courvoisier was observed.[31]

Eclipses of Jupiter's satellites
It is well known that in 1879 James Clerk Maxwell wrote to David Peck Todd asking him about the possibility of computing the velocity of the solar system through the ether using available data on occultation of Jupiter's satellites.[32] Maxwell supposed that the motion of the solar system would produce an anisotropy of the speed of light that could be detected as a fluctuation of the times of occultation of Jupiter's satellites, observed from the Earth, with a period of about 12 years. Todd answered, however, that the measurements available at that time were not precise enough for such computations.
In 1930 Courvoisier published a paper where he presented an analysis of available observations of Jupiter's satellites and claimed that they led to a new determination of the velocity of the solar system relative to the ether.[33] He used data relative to the three inner Galilean satellites published by the Johannesbourg observatory (1908-1926), comparing those measurements to those of the observatories of Cape Town, Greenwich and Leyden (1913-1924). He confirmed Maxwell's anticipation of a fluctuation with a period of about 12 years and obtained the following results:

$$A = 126° \pm 10°;\ D = +20°;\ v = 885 \pm 100 \text{ km/s}$$

Secular aberration of light
According to the theory of ether accepted by Courvoisier, the speed of light is constant relative to the ether, but could not be constant relative to the

[31] Rudolf Tomaschek and Walter Schaffernicht, "Zu den gravimetrischen Bestimmungsversuchen der absoluten Erdbewegung", *Astronomische Nachrichten*, ccxliv (1932), 257-66.
[32] James Clerk Maxwell, "On a possible mode of detecting a motion of the solar system through the luminiferous ether", *Proceedings of the royal society of London*, xxx (1879-1880), 108-10.
[33] Leopold Courvoisier, "Ableitung der 'absoluten' Erdbewegung aus beobachteten Längen der Jupiter-Satelliten", *Astronomische Nachrichten*, ccxxxix (1930), 33-38.

Earth: there should be an observable anisotropy of the speed of light due to the absolute motion of the Earth. He assumed that this would produce an observable difference in measurements of stellar aberration observed in different directions.[34] Using the available data, Courvoisier obtained the following results:

$$A = 112° \pm 20°;\ D = +47° \pm 20°;\ v = 600 \pm 305 \text{ km/s}$$

Final comments

Courvoisier's measurements of the absolute velocity of the Earth belong to the same group of Dayton Miller's and Ernest Esclangon's works.[35] However, Courvoisier's work embodied a much wider and impressive group of measurements than those of his contemporaries.

Courvoisier measured the velocity of the Earth relative to the ether using several different methods. The effects he was searching for were very small (second order in *v/c*) but the results presented were significantly larger than the estimated experimental error. The measured values of the right ascension of the Earth's motion apex varied from 52° to 126°, with a strong concentration of values between 60° and 90°. The measured declination varied between +27° and +55°, most values falling between +34° and +46°. The values obtained for the speed of the Earth varied between 300 km/s and 927 km/s, most results falling between 500 km/s and 810 km/s.

What impact did Courvoisier's work have? His researches were seldom cited. Miller and Esclangon did refer to some of his researches, because they were also reporting positive effects ascribed to the motion of the Earth through the ether. Besides those citations, there were just a few other references. General Gerold von Gleich, a well-known anti-relativist,[36] did refer to Courvoisier's results in two papers. In a short note, von Gleich mentioned fluctuations of the aberration constant that could be an indirect confirmation of Courvoisier's results.[37] In a second paper, von Gleich presented several independent confirmations of Courvoisier's measurements of the motion of the solar system.[38] He reported that Carl Wilhelm Wirtz and Gustaf Strömberg had evaluated this motion analyzing the velocities of spiral nebulae, obtaining speeds compatible with

[34] Leopold Courvoisier, "Bestimmung der absoluten Translation der Erde aus der säkularen Aberration", *Astronomische Nachrichten*, ccxli (1932), 201-12.

[35] There is a detailed historical study of Miller's work: Loyd S. Swenson, Jr., *The ethereal aether. A history of the Michelson-Morley-Miller aether-drift experiments, 1880-1930* (Austin, 1972).

[36] Joseph Wodetsky, "Gerold von Gleich", *Astronomische Nachrichten*, cclxvi (1938), 63-4.

[37] Gerold von Gleich, "Translation des Fixsternsystems und Aberrationskonstante", *Astronomische Nachrichten*, ccxli (1931), 201-02.

[38] Gerold von Gleich, "Bemerkung zur absoluten Translation unseres lokalen Fixsternsystems", *Astronomische Nachrichten*, ccxlii (1931), 273-8.

Couvoisier's results (from 630 to 820 km/s) and directions roughly compatible with his.[39] He also described his own analysis of the fluctuation of the aberration constant, and the analysis of circumpolar stars, as compatible with Courvoisier's results. His conclusion was:

> Personally, I have no doubt that the works of Mr Courvoisier, especially those on the fluctuations of the constant of aberration and those on the light speed (Jupiter's moons) prove the existence of an absolute translation of our local star system with a speed of about 600 km/s towards a point close to the ecliptic, with a longitude of about 110º. [...] Therefore, the foundations of special relativity theory are completely shattered by astronomical means.[40]

Few astronomers and physicists of that time agreed with this opinion, however. Courvoisier's researches were neither accepted, nor criticized – they were just ignored by most scientists.

Notice also that Courvoisier was a professional astronomer, and his routine measurements were always accepted and used without further questioning. Why did the scientific community ignore Courvoisier's anti-relativistic results? Several factors may have contributed to that attitude:

1. In the 1920's Einstein's theory had been successfully confirmed and most physicists and astronomers were convinced that it was the correct theory. Attempts to bring the ether again to life seemed too old-fashioned and most scientists would not be willing to hear or to read about such attempts[41].
2. Many of Courvoisier's papers were published in the *Astronomische Nachrichten*, a journal that was clearly opposed to Einstein's theory. Most scientists supporting the theory of relativity would dismiss any anti-relativist account published in that journal[42].

[39] Carl Wilhelm Wirtz, "Einiges zur Statistik der Radialbewegungen von Spiralnebeln und Kugelsternhaufen", *Astronomische Nachrichten*, ccxv (1922), 349-54; *idem*, "Die Trift der Nebelflecke", *Astronomische Nachrichten*, cciii (1916), 197-220; *idem*, "Über die Eigenbewegungen der Nebelflecke", *Astronomische Nachrichten*, cciv (1917), 23-30; Gustaf Strömberg, "Analysis of radial velocities of globular clusters and non-galactic nebulae", *Astrophysical journal*, lx (1925), 353-62.

[40] Von Gleich, "Translation des Fixsternsystems und Aberrationskonstante" (ref. 38), 278.

[41] This was also the main reason why Quirino Majorana's measurements of the absorption of gravitation and Kurt Bottlinger's explanation of the anomalies of the motion of the moon using the same assumption were dismissed by the scientific community. See Roberto de Andrade Martins, "The search for gravitational absorption in the early 20th century", in H. Goemmer, J. Renn, and J. Ritter (eds.), *The expanding worlds of general relativity* (Boston, 1999), 3-44.

[42] The editor of *Astronomische Nachrichten* from 1907 to 1938 was Hermann Kobold, who supported the publication of anti-Einstein and anti-relativistic papers,

3. Courvoisier's did not build a comprehensive theory that could be regarded as an alternative to the theory of relativity. He used a strange combination of classical physics together with the hypothesis of Lorentz's contraction, and never published a detailed derivation of his equations.[43]
4. The observed effects were very small (usually a few tenths of arc-second) and there were always large relative fluctuations of the measurements. Any single measurement published by Courvoisier could be regarded as the result of random or unknown systematic errors. The agreement between different measurements could be regarded as due to chance, or to a process of "cooking" the results.

Notice, however, that several of Courvoisier's computations were grounded upon published data obtained by other observers. Whenever Courvoisier himself made the observations, he published the data used for his computations. Anyone wishing to check his calculations could have used the available data to do so. It was not too difficult to repeat some of his observations, either.[44] It is difficult to understand why the physicists and astronomers of that time did not care to do that.

Some historical circumstances may explain, in part, the neglect of Courvoisier's researches. After the end of World War I there was a strong opposition, in Germany, to Einstein and relativity theory.[45] Everything that could be used against the theory of relativity was used – from serious scientific arguments to empty rhetoric. In this historical context, one could think that Courvoisier's work was just a biased piece of anti-Einstein propaganda, and had no scientific value. One might think that he was not a honest scientist: perhaps he falsified his data, described experiments he never made, "cooked" his results, and so on. Or maybe he was a careless scientist and just observed what he wanted to observe.

It is therefore relevant to elucidate that Courvoisier did not belong to the strong anti-relativist and anti-Einstein group of the early 1920's. He was never personally associated to Philipp Lenard and Ernst Gehrcke, for

regardless of their scientific merit. This journal published, for instance, the works of Thomas Jefferson Jackson See, that were not accepted in any other journal. *Cf.* Thomas J. Sherrill, "A career of controversy: the anomaly of T. J. J. See", *Journal for the history of astronomy*, xxx (1999), 25-50.

[43] Notice that Courvoisier's work was incompatible with Lorentz's mature ether theory, that incorporated the principle of relativity.

[44] Nowadays, it would be possible to check the reality of Courvoisier's effects using more precise routine experimental data available, and using better (computer) numerical methods. Several of his experiments could also be repeated using automatic instruments with a higher precision and in improved controlled conditions.

[45] David E. Rowe, "Einstein's allies and enemies: debating relativity in Germany, 1916-1920", in Vincent F. Hendricks, et. al. (eds.), *Interactions: mathematics, physics and philosophy, 1860-1930* (Dordrecht, 2006), 231-280; Hubert Goenner, "The reaction to relativity theory I: the anti-Einstein campaign in Germany in 1920", *Science in context*, vi (1993), 107-33; *idem*, "The reaction to relativity theory in Germany III. Hundred authors against Einstein", *Einstein studies*, v (1993), 248-73.

instance. His name was not included in the 1931 publication *Hundert Autoren gegen Einstein*.[46] Instead of irrationally opposing Einstein, he met him and exchanged letters with him for several years – without reaching any agreement, but adopting a scientific attitude.[47] Notice, also, that Courvoisier never cited the anti-Einstein scientists.

Another relevant piece of information concerns Courvoisier's political viewpoint.[48] He was strongly opposed to national socialism, and spoke about Nazis in a negative tone. He always kept his Swiss citizenship, and this helped him to keep out of the political turmoil that was going on around him. In 1943, during the World War II, he obtained permission to spend the summer vacations in Switzerland with his family, and never returned to Germany. When the war was over, the Babelsberg observatory and the house belonging to Courvoisier (built close to the observatory) became part of East Berlin. He preferred to remain in Switzerland, but suffered many difficulties, because his pension (he had retired in 1938) was not paid anymore. He lived for several years thanks to a Swiss social insurance, and to the payment he received for the edition of Euler's works. About ten years after the end of the war, West Germany began to pay his pension again.

According to Courvoisier's daughter, "He was convinced that he had found something that was true. He was convinced that this truth would find its way in the long run".[49] Leopold Courvoisier produced his research, published his data and conclusions, and expected some positive response, but he never tried hard enough to publicise his results and to convince other people that he had obtained very important results. It seems that he kept a low profile, and never attempted to join other researchers who had also obtained similar results (such as Miller or Esclangon) to produce an anti-relativist front.

Since this is the first study of Courvoisier's researches on the motion of the Earth through the ether, there is much more work to be done. It is desirable to plunge deeper into the scientific and extra-scientific features of this puzzling historical episode.

[46] *Cf.* Goenner, , "The reaction to relativity theory in Germany III. Hundred authors against Einstein" (ref. 45), 273.

[47] Courvoisier met Einstein in January 1924 and corresponded with him until October 1928, with no agreement being reached. *Cf.* Klaus Hentschel, "Einstein's attitude towards experiments", *Studies in history and philosophy of science*, xxiii (1992), 593-624, p. 613.

[48] Some personal information presented here concerning Leopold Courvoisier was obtained in an interview with his daughter Rosemarie and her husband Dietrich Ritschl, in Basel, on 31 August 1999.

[49] Rosemarie Ritschl (ref. 48).

Acknowledgements

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The Very Early History of Trigonometry

Dennis W. Duke, Florida State University

The early history of trigonometry, say for the time from Hipparchus through Ptolemy, is fairly well established, at least in broad outline (van Brummelen 2009). For these early astronomers plane trigonometry allowed the solution of an arbitrary right triangle, so that given either of the non-90° angles one could find the ratio of any two sides, or given a ratio of sides one could find all the angles. In addition the equivalent of the law of sines was known, although use infrequently, at least by Ptolemy. This skill was fully developed by the time Ptolemy wrote the *Almagest*, *ca* 150 CE (Toomer 1980), and he used it to solve a multitude of problems, some of them quite sophisticated, related to geometric models of astronomy. Ptolemy's sole tool for solving trigonometry problems was the chord: the length of the line that subtends an arc of arbitrary angle as seen from the center of a circle. Using a standard circle of radius 60, the *Almagest* gives a table of these chords for all angles between ½° and 180° in increments of ½°, and indeed Ptolemy gives a fairly detailed account of how one can compute such a table using the geometry theorems known in his time. Curiously, but not all that unusual for Ptolemy, it appears that some of the chord values in the *Almagest* were not in fact derived using the most powerful theorems that Ptolemy possessed (van Brummelen 1993, 46-73).

We also have evidence from Ptolemy that Hipparchus, working around 130 BCE, was able to solve similar trigonometry problems of about the same level of difficulty. For example, regarding finding the eccentricity and direction of apogee for the Sun's simple eccentric model, Ptolemy writes , Ptolemy writes in *Almagest* III 4:

> These problems have been solved by Hipparchus with great care. He assumes that the interval from spring equinox to summer solstice is 92½ days, and that the interval from summer solstice to autumn equinox is 92½ days, and then, with these observations as his sole data, shows that the line segment between the above-mentioned centres is approximately $\frac{1}{24}^{th}$ of the radius of the eccentre, and that the apogee is approximately 24½° in advance of the summer solstice.

The similar problem of finding the eccentricity and direction of apogee for the Moon's simple epicycle model is complicated by the moving lunar apogee. A glance at Figure 1 and a few moments consideration might give you some feel for the more advanced difficulty level of this particular problem. that Ptolemy explains in *Almagest* IV 6:

> In this first part of our demonstrations we shall use the methods of establishing the theorem which Hipparchus, as we see, used before us. We, too, using three lunar eclipses, shall derive the maximum difference from the mean motion and the epoch of the [moon's position] at the apogee, on the assumption that only this [first] anomaly is taken into account, and that it is produced by the epicyclic hypothesis.

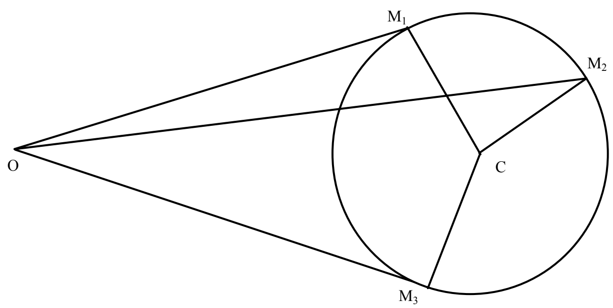

Figure 1. Consider a circle with center C and radius r. Let the distance $OC = R$. The angles M_1CM_2, M_2CM_3 and M_1OM_2, M_2OM_3 are given, and the problem is to find r/R. For a solution see *Almagest* IV 6 or Toomer 1973.

Finally, in *Almagest* IV 11 Ptolemy presents two trios of lunar eclipses that he says Hipparchus had used to determine the size of the first anomaly in lunar motion. Ptolemy gives just the results of Hipparchus' solutions, and from these we learn that while Hipparchus was certainly a capable user of trigonometry, he used a different set of numerical conventions than those used by Ptolemy. For example, while Ptolemy used a standard 360° degree circle with a radius of 60 parts, Hipparchus apparently specified the circumference of his circle as having 21,600 (= 360 x 60) parts, so that his diameter was about 6875 parts and his radius was about 3438 parts (Toomer 1973). We cannot, however, be sure whether Hipparchus used the same chord construct as Ptolemy, or perhaps just gave the ratio of side lengths corresponding to a set of angles. Nor can we be sure whether Hipparchus used a systematized table, or if he did, the angle increments of that table (Duke 2005).

One attempt to resolve these questions comes not from Greek or Roman sources, but from texts from ancient India that date from perhaps 400 – 600 CE. For many reasons, including the use of the circumference convention identical to that used by Hipparchus, and in spite of their appearance in India some six centuries after Hipparchus, it is has been proposed that these texts reflect a Greco-Roman tradition that is pre-Ptolemaic and largely

otherwise unknown to us (Neugebauer 1956, Pingree 1976,1978, van der Waerden 1961). These proposals have so far eluded definitive confirmation (and neither have any effective refutations appeared), but if they are true for the parts involving trigonometry, then it would seem plausible that Hipparchus' working set of tools included tables with 23 (non-trivial) entries of side ratios in angular increments of 3¾°, corresponding to chords in increments of 7½°, for we find exactly such tables in many Indian texts, always embedded in astronomical material that is extremely similar to early Greek astronomy.

We might be able to understand Hipparchus' use of trigonometry somewhat better if we had a little more idea how it was developed. There is a Greek source that might well be helpful in this regard, namely Archimedes' *Measurement of a Circle* (Heath 1897). Archimedes' mathematical methods in this paper are well-known: he uses the bounds

$$\tfrac{265}{153} < \sqrt{3} < \tfrac{1351}{780}$$

on $\sqrt{3}$ and then alternately circumscribes and inscribes a set of regular polygons around a circle, ultimately computing the ratio of the circumference of 96-sided polygons inside and outside the circle to the diameter of the circle, thus establishing bounds on π as

$$3\tfrac{10}{71} < \pi < 3\tfrac{1}{7}$$

What Archimedes actually computes in both cases (circumscribing and inscribing), however, are the ratios of the lengths of sides for a series of right triangles with smallest interior angle 30°, 15°, 7½°, 3¾°, (and partially $1\tfrac{7}{8}°$), and so except for normalization many of the entries for the tables used in India and perhaps also by Hipparchus are computed in Archimedes' text, and all the entries are easily found using Archimedes' method.

Thus, denoting the opposite side, the adjacent side, and the hypotenuse by *a*, *b,* and *c* Archimedes finds for the circumscribed sequence of right triangles ratios of the following values:

	a	*b*	*c*
30°	153	265	306
15°	153	571	591 1/8
7½°	153	1162 1/8	1172 1/8
3¾°	153	2334 3/8	2339 3/8

The entries in the first row result from Archimedes' lower bound on $\sqrt{3}$, while the entries in row *i*+1 follow from those in row *i* using Archimedes' algorithm:

$$a_{i+1} = a_i$$
$$b_{i+1} = b_i + c_i$$
$$c_{i+1} = \sqrt{a_{i+1}^2 + b_{i+1}^2}$$

The ratios for the complementary angles 60°, 75°, 82½°, and 86¼° are trivially obtained by interchanging columns *a* and *b*, and we now have the ratios for eight of the 23 non-trivial angles in the sequence. We may get an additional eight values by applying Archimedes' algorithm to the angles 82 ½ °, yielding the table entries for 41¼° and 48¾°, to the angle 75°, yielding the entries for 37½°, 52½°, 18¾°, and 71¼°, and to the angle 52½ °, yielding the entries for 26¼° and 63¾°. Thus we get:

	a	*b*	*c*
41¼°	1162 1/8	1324 7/8	1762 3/8
37½°	571	744	937 7/8
18¾°	571	1682	1776 1/4
26¼°	744	1508 7/8	1682 3/8

and the ratios for the complementary angles again come from interchanging *a* and *b*.

Thus 16 of the 23 table entries are immediately available directly from Archimedes' text. To get the remaining seven entries it is necessary to repeat Archimedes' analysis beginning from a 45° right triangle and bounds on $\sqrt{2}$. If Archimedes used the bounds

$$\tfrac{1393}{985} < \sqrt{2} < \tfrac{577}{408}$$

then one would find for the sequence of circumscribed triangles ratios of the following values:

	a	*b*	*c*
45°	985	985	1393
22½°	985	2378	2573 7/8
11¼°	985	4951 7/8	5049
33¾°	2378	3558 6/8	4280 1/8

and the ratios for the complimentary angles 67½°, 78¾°, 56¼° again follow from interchanging *a* and *b*.

The analysis of the inscribed triangles follows the same algorithm but instead begins with the upper bounds on $\sqrt{3}$ and $\sqrt{2}$. The resulting bounds on the ratios are so close that for all practical purposes – let us remember, these are used for analysis of measured astronomical angles, and we use linear interpolation for untabulated angles – we can use either set, or their average, with no appreciable difference in results. Here is the entire set of entries:

	circumscribed		inscribed		circumscribed	inscribed
Angle	*a*	*c*	*a*	*c*	Base 3438	Base 3438
3 6/8	153	2339 3/8	780	11926	225	225
7 4/8	153	1172 1/8	780	5975 7/8	449	449
11 2/8	985	5049	408	2091 3/8	671	671
15	153	591 1/8	780	3013 6/8	890	890
18 6/8	571	1776 2/8	2911	9056 1/8	1105	1105
22 4/8	985	2573 7/8	408	1066 1/8	1316	1316
26 2/8	744	1682 3/8	3793 6/8	8577 3/8	1520	1520
30	153	306	780	1560	1719	1719
33 6/8	2378	4280 1/8	985	1773	1910	1910
37 4/8	571	937 7/8	2911	4781 7/8	2093	2093
41 2/8	1162 1/8	1762 3/8	5924 6/8	8985 6/8	2267	2267
45	985	1393	408	577	2431	2431
48 6/8	1324 7/8	1762 3/8	6755 7/8	8985 6/8	2584	2584
52 4/8	744	937 7/8	3793 6/8	4781 7/8	2727	2727
56 2/8	3558 6/8	4280 1/8	1474 1/8	1773	2858	2858
60	265	306	1351	1560	2977	2977
63 6/8	1508 7/8	1682 3/8	7692 7/8	8577 3/8	3083	3083
67 4/8	2378	2573 7/8	985	1066 1/8	3176	3176
71 2/8	1682	1776 1/8	8575 4/8	9056 1/8	3255	3255
75	571	591 1/8	2911	3013 6/8	3320	3320
78 6/8	4951 7/8	5049	2051 1/8	2091 3/8	3371	3371
82 4/8	1162 1/8	1172 1/8	5924 6/8	5975 7/8	3408	3408
86 2/8	2334 3/8	2339 3/8	11900 4/8	11926	3430	3430

In the table above, for each angle in col. 1 cols. 2–3 and cols. 4–5 give the lengths of the opposite side and the hypotenuse for the circumscribed and inscribed triangles, respectively, in Archimedes' method. Cols. 6 and 7 give the rounded length of the opposite side assuming the hypotenuse has length 3438 parts, corresponding to a circumference of 21,600 parts. Note that for all 23 angles the ratios for each angle are identical to the level of approximation used.

Therefore, we see that using Archimedes' method, and in many cases the very numbers that appear in his text, anyone could have assembled the table in increments of 3¾° that was used in India and might have been used by Hipparchus. The two steps needed to go beyond Archimedes are (a) a normalization convention, and (b) an interpolation scheme, and there seems no reason to doubt that any competent mathematician of the time would have the slightest trouble dealing with either issue. We are certainly in no position to say that Archimedes himself constructed the table, or who in the century between Archimedes and Hipparchus did it, but it is clear that by the time of Archimedes' paper all the needed tools and results were in place, except possibly for the motivation to actually organize the table.

We can, in fact, go even farther back into the very early history of trigonometry by considering Aristarchus' *On Sizes and Distances* (Heath 1913), and we shall see that a plausible case can be made that his paper could easily have been the inspiration for Archimedes' paper. The problem Aristarchus posed was to find the ratio of the distance of the Earth to the Moon to the distance of the Earth to the Sun. He solved this problem by assuming that when that the Moon is at quadrature, meaning it appears half-illuminated from Earth and so the angle Sun-Moon-Earth is 90°, the Sun-Moon elongation is 87°, and so the Earth-Moon elongation as seen from the Sun would be 3°. Thus his problem is solved if he can estimate the ratio of opposite side to hypotenuse for a right triangle with an angle of 3°, or simply what we call sin 3°. In addition, for other problems in the same paper Aristarchus also needed to estimate sin 1° and cos 1°.

Aristarchus proceeded to solve this problem is a way that is very similar to, but not as systematic as, the method used by Archimedes. By considering circumscribed (Fig. 2) and inscribed triangles (Fig 3) and assuming a bound on $\sqrt{2}$ Aristarchus effectively establishes bounds on sin 3° as

$$\tfrac{1}{20} < \sin 3^\circ < \tfrac{1}{18}$$

and, although he does not mention it, this also establishes bounds on π as

$$3 < \pi < 3\tfrac{1}{3}$$

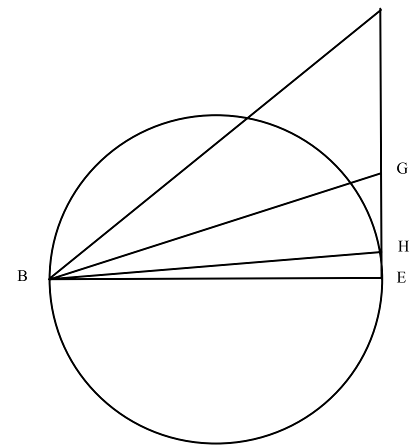

Figure 2. BE is a diameter of the circle, angle EBF is 45°, angle EBG is 22½°, and angle EBH is 3° (not to scale). Since EBG/EBH = 15/2 then GE/EH > 15/2. Since FG/GE = $\sqrt{2}$ > 7/5 then FE/EG > 12/5 = 36/15 and so FE/EH > (36/15)(15/2) = 18/1.

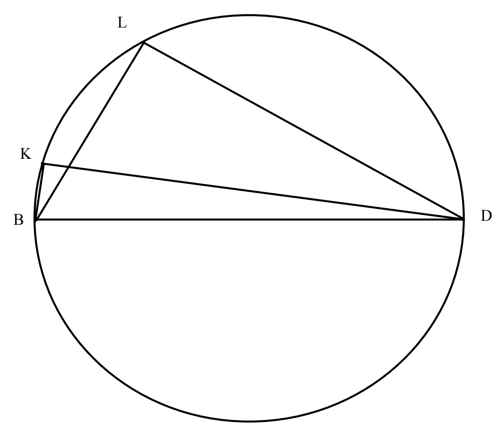

Figure 3. BD is a diameter of the circle, angle BDL = 30°, and angle BDK = 3° (not to scale). Since arc BL = 60° and arc BK = 6° then BL/BK < 10/1. Since BD = 2 BL then BD/BK < 20/1.

Later, in Propositions 11 and 12 Aristarchus proves using similar methods that

$$\tfrac{1}{60} < \sin 1^\circ < \tfrac{1}{45}$$

and

$$\tfrac{89}{90} < \cos 1^\circ < 1$$

always understanding, of course, that what we write as sine and cosine was to Aristarchus a ratio of sides in a right triangle. None of these bounds are particularly tight, and it is difficult to know if this was the best Aristarchus could do, or whether it was simply adequate for his purposes, which is apparently the case in any event.

The similarities between Aristarchus' and Archimedes' methods are clear: both assume bounds on a small irrational number, and hence effectively on the value of sin α for some relatively large angle, 60° or 45°, and through a sequence of circumscribed and inscribed triangles on a circle establish bounds on a target small angle, 3° for Aristarchus and $1\tfrac{7}{8}^\circ$ for Archimedes. Archimedes clearly realizes that this established bounds on π; Aristarchus may or may not have realized it, or might have not considered his bounds interesting enough to mention. Both Aristarchus and Archimedes are focused firmly on the relations between angles and ratios of sides in right triangles, neither ever using anything related to the chord construct used by Ptolemy. We know that Archimedes and Aristarchus exchanged correspondence, and we know that Archimedes was well aware of Aristarchus' work on the Earth–Moon–Sun distance problem. Indeed, Archimedes tells us that his own father also worked on the problem. In any case the parallels in the two calculations are quite striking, and it is not hard to imagine that Aristarchus' calculation could have been the inspiration behind Archimedes' calculation.

Coupled with the fact that the sin and not the chord is used also in the Indian texts, this suggests that the chord was introduced later rather than sooner, and certainly offers no encouragement to anyone claiming that Hipparchus used chords or that the sine was invented in India as an 'improvement' over the chord.

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An Early Use of the Chain Rule

Dennis W Duke, Florida State University

One of the most useful tools we learned when we were young is the chain rule of differential calculus: if $q(\alpha)$ is a function of α, and $\alpha(t)$ is a function of t, then the rate of change of q with respect to t is

$$\frac{dq}{dt} = \frac{dq}{d\alpha} \cdot \frac{d\alpha}{dt}$$

In the special case that $a(t)$ is linear in t, so $\alpha(t) = \alpha_0 + \omega_a (t - t_0)$, this becomes

$$\frac{dq}{dt} = \frac{dq}{d\alpha}\omega_a$$

If $q(\alpha)$ is a complicated function of α, for example

$$q(\alpha) = \tan^{-1}\left(\frac{-e\sin\alpha}{R + e\cos\alpha}\right)$$

then the computation of $dq/d\alpha$ is not necessarily easy. In this case

$$\frac{dq}{d\alpha} = \frac{-e/R\cos\alpha - (e/R)^2}{1 + 2e/R\cos\alpha + (e/R)^2}$$

so when e/R is small we have simply

$$\frac{dq}{d\alpha} \simeq -\frac{e}{R}\cos\alpha$$

In cases like this a practical alternative is to tabulate $q(\alpha)$ at small intervals $\Delta\alpha$ and then estimate $dq/d\alpha$ as a ratio of finite differences:

$$\frac{dq(\alpha)}{d\alpha} \simeq \frac{q(\alpha + \Delta\alpha) - q(\alpha)}{\Delta\alpha}$$

This particular function $q(\alpha)$ in our example is, of course, the equation of center for the simple eccentric (or, equivalently, epicycle) model used by Hipparchus and later Ptolemy for the Sun and the Moon (at syzygy), and it connects the mean longitude $\bar{\lambda}$ and true longitude λ according to

$$\lambda = \bar{\lambda} + q(\alpha)$$

where $\alpha = \bar{\lambda} - A$ and A is the longitude of apogee. As we shall see, Ptolemy very clearly knew that the rate of change with time of the true longitude λ is

$$\frac{d\lambda}{dt} = \omega_t + \omega_a \frac{dq}{d\alpha}$$

where ω_t and ω_a are the mean motion of the Moon in longitude and anomaly. Actually proving the chain rule is straightforward enough, but not entirely trivial, although perhaps in this simple case it might be guessed by dimensional analysis. As is often the case, Ptolemy gives no hint of how he came to know it.

It is, I think, not as widely appreciated as it might be that the result just given appears in Ptolemy's *Almagest*, not once but twice, and so was known at least as early the 2nd century CE, and very probably was known to Hipparchus in the 2nd century BCE, therefore nearly two millennia before the development of differential calculus (for standard treatments see, e.g. Neugebauer 1975, 122-124, 190-206 or Pedersen 1974, 225-226, 341-343).

The first occurrence of this result is found in *Almagest* VI 4. Ptolemy has just completed explaining how to compute the time $\bar{t}$ of some mean syzygy – a conjunction or opposition of the Sun and Moon in mean longitude – using their known mean motions and epoch positions in mean longitude and anomaly, and is ready to show how to estimate the time $t = \bar{t} + \delta t$ of the corresponding true syzygy. Therefore let us consider the case of a mean conjunction at some time $\bar{t}$, so that

$$\bar{\lambda}_S(\bar{t}) = \bar{\lambda}_M(\bar{t})$$

and work out what Ptolemy would do if he knew calculus.

Since we know the mean anomalies $\alpha_S(\bar{t})$ and $\alpha_M(\bar{t})$ at time $\bar{t}$ we can also compute the equations $q_S(\alpha(\bar{t}))$ and $q_M(\alpha(\bar{t}))$. At time t of true syzygy we have

$$\bar{\lambda}_S(t) + q_S(\alpha_S(t)) = \bar{\lambda}_M(t) + q_M(\alpha_M(t))$$

(with, of course, the addition of 180° on one side of the equation in the case of an opposition). Since the mean longitudes vary linearly in time we have simply

$$\bar{\lambda}_M(t) = \bar{\lambda}_M(\bar{t} + \delta t) = \bar{\lambda}_M(\bar{t}) + \omega_t \delta t$$
$$\bar{\lambda}_S(t) = \bar{\lambda}_S(\bar{t} + \delta t) = \bar{\lambda}_S(\bar{t}) + \omega_S \delta t$$

where ω_S is the mean motion of the Sun, so that

$$\bar{\lambda}_M(t) - \bar{\lambda}_S(t) = (\omega_t - \omega_S)\delta t = \eta \delta t = q_S(\alpha_S(t)) - q_M(\alpha_M(t))$$

Furthermore, since δt is small compared to the orbital period of the Moon, and even more so the Sun, we have

$$q_M(\alpha_M(t)) = q_M(\alpha_M(\bar{t})) + \delta t \left.\frac{dq_M}{dt}\right|_{t=\bar{t}} + O(\delta t^2)$$
$$\simeq q_M(\alpha_M(\bar{t})) + \omega_a \delta t \left.\frac{dq_M}{d\alpha}\right|_{t=\bar{t}}$$
$$q_S(\alpha_S(t)) = q_S(\alpha_S(\bar{t})) + \delta t \left.\frac{dq_S}{dt}\right|_{t=\bar{t}} + O(\delta t^2)$$
$$\simeq q_S(\alpha_S(\bar{t})) + \omega_S \delta t \left.\frac{dq_S}{d\alpha}\right|_{t=\bar{t}}$$

noting that for the standard solar model of Hipparchus and Ptolemy the mean motions in longitude and anomaly of the Sun are equal since the solar apogee is tropically fixed.

Combining these and solving for δt gives

$$\delta t = \frac{q_S(\alpha_S(\bar{t})) - q_M(\alpha_M(\bar{t}))}{\eta + \omega_a \left.\frac{dq_M}{d\alpha_M}\right|_{t=\bar{t}} - \omega_S \left.\frac{dq_S}{d\alpha_S}\right|_{t=\bar{t}}}$$

Ptolemy, of course, does not know how to do a Taylor expansion approximation, but the result he gives is uncannily similar. First he instructs

us to estimate the true distance between the Sun and Moon at mean syzygy, which we see from the above is

$$q_S(\alpha_S(\bar{t})) - q_M(\alpha_M(\bar{t}))$$

He then says to multiply this by $^{13}\!/_{12}$ and to divide that result by the Moon's true speed, which he estimates as

$$0;32,56^{\circ/hr} - 0;32,40^{\circ/hr}(\text{q}(\alpha + 1^\circ) - q(\alpha))$$

where 0;13,56°/hr is the Moon's mean motion in longitude ω_t expressed in degrees per equinoctial hour, and similarly 0;32,40°/hr is the hourly mean motion in anomaly. Note also that

$$q(\alpha + 1^\circ) - q(\alpha) = \frac{q(\alpha + 1^\circ) - q(\alpha)}{1^\circ} = \left.\frac{\Delta q}{\Delta \alpha}\right|_{t=\bar{t}}$$

so Ptolemy has estimated *dq/dα* with a finite difference approximation, and furthermore chosen an interval $\Delta\alpha = 1°$ that, at first sight, cleverly avoids an otherwise bothersome division operation.

So in the end his estimate of the correction δt to the mean time $\bar{t}$ is, in units of equinoctial hours,

$$\delta t = \frac{q_S(\alpha_S(\bar{t})) - q_M(\alpha_M(\bar{t}))}{\frac{12}{13}\left(0;32,56^\circ + 0;32,40^\circ \left.\frac{dq_M}{d\alpha_M}\right|_{t=\bar{t}}\right)}$$

which compares very closely to the more exact result derived above, the only differences being that he has two approximations in the denominator: first, he gives

$$\frac{12}{13} \times 0;32,56 = 0;30,24$$

which is a good approximation to η = 0;30,8, and second he neglects the term proportional to $dq_S/d\alpha_S$ which is smaller than the already small (compared to 0;32,56) derivative of the Moon's anomalistic equation of center.

Although Ptolemy's scheme of estimating $dq/d\alpha \simeq q(\alpha + 1^\circ) - q(\alpha)$ is certainly one option, it is not necessarily the best option when the task is to make the estimate using a table of $q(\alpha)$ values, especially the table found in the *Almagest*, where the table entries are either 3° or 6° apart.. For one reason, it requires two table interpolations. Yet these can be easily avoided if the instructions are instead to find the interval in which α lies, i.e. find α_i and α_{i+1} such that $\alpha_i \le \alpha < \alpha_{i+1}$ (which can be done by inspection), and then estimate $dq/d\alpha$ using

$$\frac{dq(\alpha)}{d\alpha} = \frac{q(\alpha_{i+1}) - q(\alpha_i)}{\alpha_{i+1} - \alpha_i}$$

which, given the piecewise linearity of the table, is about the best estimate you can make in any case without resorting to a higher order interpolations scheme. Furthermore, the quotients on the right hand side of the above equation could all be precomputed and included in the table and would be useful for all table interpolations, but that is not done in the *Almagest*. Thus, the procedure that Ptolemy describes would make a lot more sense, especially in terms of computational efficiency, if the table was compiled with an interval of 1° in the variable α. Strabo tells us that for geography Hipparchus did compile length of the longest day at intervals of 1° in terrestrial latitude, so it would not be too surprising if Hipparchus had 1° tables for lunar, and for that matter, solar anomaly.

Ptolemy goes on to estimate how close to the nodes the Moon has to be before an eclipse is even possible. For lunar eclipses this is straightforward, but for solar eclipses a rather involved calculation involving lunar parallax is required, lunar parallax having already been analyzed in detail in *Almagest* V 17–19. Ptolemy then discusses the allowed intervals (in months) between lunar and solar eclipses. Besides the common six month interval, it turns out that lunar eclipses can also occur at five month, but not seven month, intervals, and solar eclipses can occur at not only both five and seven month intervals, but also at one month intervals, provided the observers are at widely different locations, including being in different (north and south) hemispheres.

Related to all this is a passage in Pliny's *Natural History*, written *ca*. 70 CE, which says

> It was discovered two hundred years ago, by the sagacity of Hipparchus, that the moon is sometimes eclipsed after an interval of five months, and the sun after an interval of seven; also, that he becomes invisible, while above the horizon, twice in every thirty days, but that this is seen in different places at different times.

For Hipparchus to know all this, and in particular the part about solar eclipses at one month intervals, requires that he had a significant amount of computational skill, including a reasonable command of lunar parallax. Indeed, Ptolemy tells us that Hipparchus wrote two books on parallax. Therefore it is hardly a stretch to presume, with Neugebauer 1975, 129 and Pedersen 1974, 204, that Hipparchus already knew the eclipse material reported by Ptolemy in the *Almagest*, including the use of the chain rule discussed above.

Besides using the instantaneous speed to estimate the time difference between mean and true syzygy, it is also needed to estimate for lunar eclipses the time difference between first and last contact with the Earth's shadow, and in the case of total lunar eclipses, the time interval of complete immersion (and, of course, similarly for solar eclipses).

The second occurrence of the use of the chain rule is in *Almagest* VII 2 concerning retrograde motion. Ptolemy begins by recalling Apollonius' treatment (from perhaps 180 BCE) of the simple epicycle model, in which the distance from the Earth to the epicycle center is constant. The ratio of a particular pair of geometric distances is, according to Apollonius' theorem, equal to the ratio of the speed ω_t of the epicycle center to the speed ω_a of the planet on the epicycle, both of which are constant in the simple model. However, in the case of the more complicated *Almagest* planetary models – the equant for Saturn, Jupiter, Mars, and Venus and the crank mechanism for Mercury – the relevant ratio is between the true speeds v_t and v_a as observed from Earth, which are not constant, and this once again involves using the chain rule, just as above:

$$\frac{d\lambda_t / dt}{d\lambda_a / dt} = \frac{\omega_t + \frac{dq}{dt}}{\omega_a + \frac{dq}{dt}} = \frac{\omega_t + \omega_t' \frac{dq}{d\alpha}}{\omega_a + \omega_t' \frac{dq}{d\alpha}}$$

where ω_t' is ω_t diminished by $1^{\circ/cy}$ to account for the sidereally fixed apogees in the *Almagest* planetary models. In this case Ptolemy does not actually explain how to compute the numerical derivatives for $dq/d\alpha$, but the numerical values he gives for each planet confirm that he was using the tables of mean anomaly in *Almagest* XI 11, or something pretty close to them.

Returning now to eclipses, the natural question to wonder about is whether this careful estimate of the instantaneous speed is worth the effort? For example, how much difference would it make in eclipse predictions if in the calculations the mean speed η was used instead of the accurately calculated speed? In order to investigate this questions I have computed, using the *Almagest* rules, all 977 lunar eclipses from –746 to –130.

The speed is used two ways. First, it is used to compute the difference in time between mean and true conjunction, the eclipse being taken to occur at true conjunction rather than at minimum distance from the shadow center. This latter approximation is a good one, the time difference between true conjunction and minimum distance averaging less than 2 minutes and never exceeding 6 minutes, no matter which speed, mean or instantaneous, is used. On the other hand, the estimates of the actual time of true conjunction vary by about 19 minutes on average, and for about 40% of lunar eclipses the time difference exceeds 20 minutes, with a maximum difference of about 48 minutes.

Second, the speed is used to compute the duration of partial and total eclipse. Considering just partial eclipses, which are probably the easiest to time and show the largest effect in any event, the average difference in computed duration is about 12 minutes. and for about 14% of lunar eclipses the difference of computed duration of partial eclipse time interval exceeds 20 minutes, with a maximum difference of about 41 minutes. The differences that exceed 20 minutes arise when the eclipses have low magnitude, so that a relatively small change in the latitude of the Moon can result in a relatively large change in the path length needed to cross the shadow.

Altogether then, it seems reasonable to me that these differences in predicted absolute time and duration of lunar eclipses, while not exactly dramatic, are large enough to suggest a motivation for the ancient astronomer to compute the times using the instantaneous rather than the mean speed.

All of this by no means implies that differential calculus as we know it was understood by ancient mathematicians, but it does show that when they needed to solve a special problem, such as the one above, they were in some cases able to do it.

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