Numerical simulation of a physical stochastic signal in ether-drift experiments

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Abstract

Ether-drift experiments are the only known experiments which, in principle, can distinguish Einstein's interpretation of the relativistic effects from the Lorentzian point of view with some underlying form of ether which plays the role of a preferred reference frame. To test experimentally this latter point of view, it is usually assumed that the macroscopic Earth's motion should be detectable in the laboratory from the time dependence of the data. Therefore any observed stochastic signal, which does not exhibit the smooth modulations expected from the Earth's rotation, tends to be considered as a spurious instrumental effect, e.g. thermal noise. The real situation, however, might be more subtle if the hypothetical ether (i.e. the physical vacuum) resembles a turbulent fluid where large-scale and small-scale motions are only indirectly related. In this case in fact, besides thermal noise, the data might contain a genuine physical stochastic component. To test this scenario, we have performed a numerical simulation to estimate the signal in a toy model of a statistically isotropic and homogeneous turbulent flow. After subtracting the known forms of disturbances, the present data are consistent with velocity fluctuations whose absolute scale is determined by the Earth's cosmic motion with respect to the CMB (projected in the plane of the interferometer at the latitude of the laboratory). Therefore the Earth's motion, although undetectable from the naive time-dependence of the data, could nevertheless show up in their statistical distributions. In particular, the predicted non-gaussian nature of the distribution of the instantaneous data will be precisely tested with the forthcoming generation of precise cryogenic experiments, with potentially important implications for our understanding of both gravity and relativity.

1. Introduction

Ether-drift experiments, starting from the original Michelson-Morley experiment, have played a fundamental role in the history of physics. Today, they are still crucial in view of the basic ambiguity which persists in the interpretation of the relativistic effects. In fact, the basic Lorentz transformations, rather than originating from the relative motion of a pair of observers S' and S'', as in Einstein's relativity, might be associated with their *individual* velocity parameters $\beta' = v'/c$ and $\beta'' = v''/c$ [1, 2, 3] and thus reflect the existence of some underlying ethereal medium. Still, due to the fundamental group properties, the two frames S' and S'' would also be mutually connected by a Lorentz transformation with relative velocity parameter

$$\beta_{\rm rel} = \frac{\beta' - \beta''}{1 - \beta'\beta''} \equiv \frac{v_{\rm rel}}{c} \tag{1}$$

(we restrict for simplicity to one-dimensional motions). In spite of the deep conceptual difference, this would produce a substantial quantitative equivalence with Einstein's formulation for most standard experimental tests, where one just compares the relative measurements of a pair of observers. Thus, to resolve the ambiguity, one should also consider the ether-drift experiments.

At the same time, if the speed of light in the vacuum measured on the Earth's surface, say c_γ, coincides with the basic parameter c entering Lorentz transformations, relativistic effects conspire to make undetectable the individual velocity parameters β', β'',...Therefore the only possibility is that c_γ and c do not coincide *exactly*, see e.g. [4]. In this case, in fact, there would be a tiny ether-drift effect $\delta \sim \beta^2(c - c_\gamma)/c$. We emphasize that this possibility, explored in ref.[5] in the framework of the so called emergent-gravity scenario [6, 7], where $(c - c_\gamma)/c$ was estimated [5] to be $\mathcal{O}(10^{-9})$, is not ruled out by the present experimental data. In fact the ether-drift, as measured from the fractional beat signal between two vacuum optical resonators [8], gives $\delta \sim 10^{-15}$ and thus could indicate an absolute Earth's velocity of about 300 km/s, as in most cosmic motions. This basic point can be easily checked by looking at Fig. 9(a) of ref.[9] where a typical sequence of 40 data collected at regular steps of 1 second is reported [1]. As one can see, this instantaneous signal exhibits random fluctuations of about ± 1 Hz and this value, for a laser frequency of $2.82 \cdot 10^{14}$ Hz, might correspond to a genuine ether-drift effect of about $\pm 3.5 \cdot 10^{-15}$.

Still, so far, all experimental groups tend to interpret this type of stochastic signal as a mere instrumental effect, e.g. thermal noise. However, as discussed in [5], this interpretation

[1] With respect to other articles, ref.[9] has the advantage to report the instantaneous raw data. The experiment also adopts a sophisticated geometrical set-up where, to minimize all possible asymmetries, the two optical cavities are obtained from the same block of ULE (Ultra Low Expansion) material. As such, the results of ref.[9] will play an important role in our analysis.

is not so obvious. For instance the same typical magnitude 10^{-15} was obtained in [5] from an independent re-analysis of the cryogenic experiment of ref.[10]. Since thermal noise is estimated on the basis of the fluctuation-dissipation theorem, there is no real reason that both room temperature and cryogenic experiments exhibit the same instrumental effects.

To explore this important aspect, in this paper, we shall adopt the same point of view of ref.[5]. Namely, besides thermal noise (which is expected any way in all room-temperature experiments) there might be a genuine physical stochastic component in the data as, for instance, if the physical vacuum were similar to a form of highly turbulent ether. This idea is deep rooted in the basic foundational aspects of both quantum theory and relativity (see [11]) and finds additional motivations in those representations of the vacuum as a form of 'space-time foam' which indeed resembles a turbulent fluid [12, 13, 14, 15, 16]. For this reason, within a simple model of statistically isotropic and homogeneous turbulence, we have performed a numerical simulation to estimate the effects that could be detected in the next generation [17] of precise cryogenic experiments.

After this general introduction, the plan of the paper is as follows. In Sect.2 we shall remind the problem of measuring the two-way velocity of light and in Sect.3 the basics of the present ether-drift experiments. In Sect.4 we shall describe our numerical simulation and give definite predictions for the next series of experiments. Finally Sect.5 will contain a summary and our conclusions.

2. The two-way velocity of light

The basic concept in ether-drift experiments is the two-way velocity of light in the vacuum $\bar{c}_\gamma(\theta)$. This is defined in terms of the one-way velocity $c_\gamma(\theta)$ (which is not unambiguously measurable) through the relation

$$\bar{c}_\gamma(\theta) \quad = \quad \frac{2c_\gamma(\theta)c_\gamma(\pi+\theta)}{c_\gamma(\theta)+c_\gamma(\pi+\theta)} \tag{2}$$

and could exhibit a non-zero anisotropy

$$\frac{\Delta\bar{c}_\theta}{c} = \frac{\bar{c}_\gamma(\pi/2+\theta)-\bar{c}_\gamma(\theta)}{\langle\bar{c}_\gamma\rangle} \neq 0 \tag{3}$$

This theoretical concept is related to the measurable frequency shift, i.e. the beat signal, $\Delta\nu$ of two optical resonators [8] through the relation

$$\delta(t) \equiv \frac{\Delta\bar{c}_\theta(t)}{c} = \frac{\Delta\nu^{\text{phys}}(t)}{\nu_0} \tag{4}$$

where ν_0 is the reference frequency of the two optical resonators and the suffix "phys" indicates a hypothetical physical part of the frequency shift after subtraction of all spurious effects.

As a possible theoretical framework for a non-zero anisotropy, we shall concentrate on a scenario which introduces some difference with respect to standard General Relativity and has a very simple motivation: $\bar{c}_\gamma$ might differ from the basic parameter c entering Lorentz transformations due to gravitational effects. To this end, as anticipated in the Introduction, one can consider the emergent-gravity scenario [6, 7] where the space-time curvature observed in a gravitational field becomes an effective phenomenon in flat space, analogously to a hydrodynamic description of moving fluids. In this perspective, gravity produces local modifications of the basic space-time units which are known, see e.g. [18, 19], to represent an alternative way to introduce the concept of curvature [2]. This scenario represents the simplest modification of the standard picture which allows for a non-vanishing anisotropy and gives the correct order of magnitude $\delta \sim 10^{-15}$. As such, it will be adopted in the rest of this paper.

To better appreciate the possible implications for the measurement of the speed of light, it is convenient to start, as in ref. [5], from the basic notion: the definition of speed as (distance moved)/(time taken). To this end, one has to choose some standards of distance and time and different choices can give different answers. Therefore, we shall adopt the *same* point of view of special relativity: the right space-time units are those for which the two-way velocity of light in the vacuum $\bar{c}_\gamma$, when measured in an inertial frame, coincides with the basic parameter c entering Lorentz transformations. However, inertial frames are just an idealization. Therefore the appropriate realization is to assume *local* standards of distance and time such that the identification $\bar{c}_\gamma = c$ holds as an asymptotic relation in the physical conditions which are as close as possible to an inertial frame, i.e. *in a freely falling frame* (at least by restricting to a space-time region small enough that tidal effects of the external gravitational potential $U_{\rm ext}(x)$ can be ignored). This is essential to obtain an operative definition of the otherwise unknown parameter c. With these premises, light propagation for an observer S' sitting on the Earth's surface can be described with increasing degrees of approximations [5, 11]:

i) In a zeroth-order approximation, S' is considered a freely falling frame. This amounts to assume $c_\gamma = c$ so that, given two events which, in terms of the local space-time units of S', differ by (dx, dy, dz, dt), light propagation is described by the condition (ff='free-fall')

$$(ds^2)_{\rm ff} = c^2dt^2 - (dx^2 + dy^2 + dz^2) = 0 \tag{5}$$

[2]This point of view has been vividly represented by Thorne in one of his books [20]: "Is space-time really curved ? Isn't conceivable that space-time is actually flat, but clocks and rulers with which we measure it, and which we regard as perfect, are actually rubbery ? Might not even the most perfect of clocks slow down or speed up and the most perfect of rulers shrink or expand, as we move them from point to point and change their orientations ? Would not such distortions of our clocks and rulers make a truly flat space-time appear to be curved ? Yes".

ii) However, is really the Earth a freely-falling frame ? To a closer look, in fact, an observer S' placed on the Earth's surface can only be considered a freely-falling frame up to the presence of the Earth's gravitational field. Its inclusion leads to tiny deviations from the standard Eq.(5). These can be estimated by considering S' as a freely-falling frame (in the same external gravitational field described by $U_{\rm ext}(x)$) that however is also carrying on board a heavy object of mass M (the Earth's mass itself) that affects the effective local space-time structure, see fig.1 of ref.[11]. To derive the required correction, let us again denote by (dx, dy, dz, dt) the local space-time units of the freely-falling observer S' in the limit $M = 0$ and by δU the extra Newtonian potential produced by the heavy mass M at the experimental set up where one wants to describe light propagation. In a flat-space interpretation, light propagation for the S' observer can then be described by the condition

$$(ds^2)_{\delta \rm U} = \frac{c^2 d\hat{t}^2}{\mathcal{N}^2} - (d\hat{x}^2 + d\hat{y}^2 + d\hat{z}^2) = 0 \tag{6}$$

where, to first order in δU, the space-time units ($d\hat{x}$, $d\hat{y}$, $d\hat{z}$, $d\hat{t}$) are related to the corresponding ones (dx, dy, dz, dt) for $\delta U = 0$ through an overall re-scaling factor

$$\lambda = 1 + \frac{|\delta U|}{c^2} \tag{7}$$

and we have also introduced a vacuum refractive index [3]

$$\mathcal{N} = 1 + 2\frac{|\delta U|}{c^2} \tag{8}$$

Therefore, to this order, light is formally described as in General Relativity where one finds the weak-field, isotropic form of the metric

$$(ds^2)_{\rm GR} = c^2 dT^2 (1 - 2\frac{|U_{\rm N}|}{c^2}) - (dX^2 + dY^2 + dZ^2)(1 + 2\frac{|U_{\rm N}|}{c^2}) \equiv c^2 d\tau^2 - dl^2 \tag{9}$$

In Eq.(9) U_N denotes the Newtonian potential and (dT, dX, dY, dZ) arbitrary coordinates defined for $U_{\rm N} = 0$. Finally, $d\tau$ and dl denote the elements of proper time and proper length in terms of which, in General Relativity, one would again deduce from $ds^2 = 0$ the same universal value $c = \frac{dl}{d\tau}$. This is the basic difference with Eqs.(6)-(8) where the physical unit of length is $\sqrt{d\hat{x}^2 + d\hat{y}^2 + d\hat{z}^2}$, the physical unit of time is $d\hat{t}$ and instead a non-trivial refractive index $\mathcal{N}$ is introduced. For an observer placed on the Earth's surface, its value is

$$\mathcal{N} - 1 \sim \frac{2G_N M}{c^2 R} \sim 1.4 \cdot 10^{-9} \tag{10}$$

[3] A general isotropic metric $(A, -B, -B, -B)$ depends on two functions which, in a flat-space picture, can be interpreted in terms of an overall re-scaling of the space-time units and of a refractive index. Since physical units of time scale as inverse frequencies, and the measured frequencies $\hat{\omega}$ for $\delta U \neq 0$ are red-shifted when compared to the corresponding value ω for $\delta U = 0$, this fixes the value of λ. Furthermore, independently of the specific underlying mechanisms, the two functions A and B can be related through the general requirement $AB = 1$ which expresses the basic property of light of being, at the same time, a corpuscular and undulatory phenomenon [21]. This fixes the value of $\mathcal{N}$.

where G_N is Newton's constant and M and R are respectively the Earth's mass and radius.

iii) Differently from General Relativity, in a flat-space interpretation with re-scaled units $(d\hat{x},\ d\hat{y},\ d\hat{z},\ d\hat{t})$ and $\mathcal{N} \neq 1$, the speed of light in the vacuum c_γ no longer coincides with the parameter c entering Lorentz transformations. Therefore, as a general consequence of Lorentz transformations, an isotropic propagation as in Eq.(6) can only be valid for a special state of motion of the Earth's laboratory. This provides the operative definition of a preferred reference frame Σ while for a non-zero relative velocity $\mathbf{V}$ one expects off diagonal elements $g_{0i} \neq 0$ in the effective metric and a tiny light anisotropy. As shown in Ref.[5], to first order in both $(\mathcal{N}-1)$ and V/c one finds

$$g_{0i} \sim 2(\mathcal{N}-1)\frac{V_i}{c} \tag{11}$$

These off diagonal elements can be imagined as being due to a directional polarization of the vacuum induced by the now moving Earth's gravitational field and express the general property [22] that any metric, locally, can always be brought into diagonal form by suitable rotations and boosts. In this way, by introducing $\beta = V/c$, $\kappa = (\mathcal{N}^2 - 1)$ and the angle θ between $\mathbf{V}$ and the direction of light propagation, one finds, to $\mathcal{O}(\kappa)$ and $\mathcal{O}(\beta^2)$, the one-way velocity [5]

$$c_\gamma(\theta) = \frac{c}{\mathcal{N}}\ \left[1 - \kappa\beta\cos\theta - \frac{\kappa}{2}\beta^2(1+\cos^2\theta)\right] \tag{12}$$

and a two-way velocity of light

$$\begin{aligned}\bar{c}_\gamma(\theta) &= \frac{2c_\gamma(\theta)c_\gamma(\pi+\theta)}{c_\gamma(\theta)+c_\gamma(\pi+\theta)} \\ &\sim \frac{c}{\mathcal{N}}\ \left[1 - \beta^2\left(\kappa - \frac{\kappa}{2}\sin^2\theta\right)\right]\end{aligned} \tag{13}$$

This allows to define the RMS [23, 24] anisotropy parameter $\mathcal{B}$ through the relation

$$\frac{\Delta\bar{c}_\theta}{c} = \frac{\bar{c}_\gamma(\pi/2+\theta) - \bar{c}_\gamma(\theta)}{\langle\bar{c}_\gamma\rangle} \sim \mathcal{B}\frac{V^2}{c^2}\cos(2\theta) \tag{14}$$

with

$$|\mathcal{B}| \sim \frac{\kappa}{2} \sim \mathcal{N}-1 \tag{15}$$

From the previous analysis, by replacing the value of the refractive index Eq.(10) and adopting, as a rough order of magnitude, the typical value of most cosmic motions $V \sim 300$ km/s, one expects a tiny fractional anisotropy

$$\frac{\langle\Delta\bar{c}_\theta\rangle}{c} \sim |\mathcal{B}|\frac{V^2}{c^2} = \mathcal{O}(10^{-15}) \tag{16}$$

that could finally be detected in the present, precise ether-drift experiments.

3. The present ether-drift experiments

Let us now consider in more detail the experimental aspects. To increase the statistics, the present experiments exhibit rotating optical resonators. For instance, the relative frequency shift for the experiment of ref.[25] can be expressed as

$$\frac{\Delta\nu^{\rm phys}(t)}{\nu_0} = S(t)\sin 2\omega_{\rm rot}t + C(t)\cos 2\omega_{\rm rot}t \tag{17}$$

where $\omega_{\rm rot}$ is the rotation frequency of one resonator with respect to the other which is kept fixed in the laboratory and oriented north-south. Notice that, by considering a fully symmetric apparatus one should introduce an overall factor of two on the right hand side of the above equantion, at least if one wants to express the measured shift in terms of the functions $S(t)$ and $C(t)$ extracted from the non-symmetric apparatus of ref.[25]. Notice also that in some articles the function $S(t)$ is denoted as $B(t)$.

In this framework, the existence of possible time modulations of the signal that might be synchronous with the Earth's rotation has always represented a crucial ingredient for the analysis of the data. This expectation derives from a model where one assumes a *fixed* preferred frame Σ. Then, for short-time observations of 1-2 days, the time dependence of a hypothetical physical signal can only be due to (the variations of the projection of the Earth's velocity $\mathbf{V}$ in the interferometer's plane caused by) the Earth's rotation. In this case, the two functions $S(t)$ and $C(t)$ admit the simplest Fourier expansion [25] ($t' = \omega_{\rm sid}t$ is the sidereal time of the observation in degrees)

$$S(t) = S_0 + S_{s1}\sin t' + S_{c1}\cos t' + S_{s2}\sin(2t') + S_{c2}\cos(2t') \tag{18}$$

$$C(t) = C_0 + C_{s1}\sin t' + C_{c1}\cos t' + C_{s2}\sin(2t') + C_{c2}\cos(2t') \tag{19}$$

with *time-independent* C_k and S_k Fourier coefficients.

This theoretical framework, accepted so far by all experimental groups, leads to average the various C_k and S_k obtained from fits performed during a 1-2 day observation period. By further averaging over many short-period experimental sessions, the data support the general conclusion [26, 27, 28] that, although the typical instantaneous $S(t)$ and $C(t)$ are indeed $\mathcal{O}(10^{-15})$, the global averages $(C_k)^{\rm avg}$ and $(S_k)^{\rm avg}$ for the Fourier coefficients are much smaller, at the level $\mathcal{O}(10^{-17})$, and, with them, the derived parameters entering the phenomenological SME [29] and RMS models.

However, there might be different types of ether-drift where the straightforward parameterizations Eqs.(18), (19) and the associated averaging procedures are *not* allowed. In fact, before assuming any definite theoretical scenario, one should first ask: if light were really propagating in a physical medium, an ether, and not in a trivial empty vacuum, how should the motion of (or in) this medium be described? Namely, could this relative motion exhibit variations that are *not* only due to known effects as the Earth's rotation and orbital revolution? As discussed in [5], by representing the physical vacuum as a fluid, the standard assumption of smooth sinusoidal variations of the signal, associated with the Earth's rotation (and its orbital revolution), corresponds to assume the conditions of a pure laminar flow associated with simple regular motions. Instead, by adopting the model of an underlying turbulent medium there might be other forms of time modulations. In this alternative scenario, the same basic experimental data might admit a different interpretation and a definite instantaneous signal $\Delta\nu(t) \neq 0$ could become consistent with $(C_k)^{\rm avg} \sim (S_k)^{\rm avg} \sim 0$.

To discuss this alternative scenario, it is convenient to first re-write Eq.(17) as

$$\frac{\Delta\nu^{\rm phys}(t)}{\nu_0} = A(t)\cos(2\omega_{\rm rot}t - 2\theta_0(t)) \tag{20}$$

where

$$C(t) = A(t)\cos 2\theta_0(t) \qquad\qquad S(t) = A(t)\sin 2\theta_0(t) \tag{21}$$

so that

$$A(t) = \sqrt{S^2(t) + C^2(t)} \tag{22}$$

Here $\theta_0(t)$ represents the instantaneous direction of a hypothetical ether-drift effect in the x-y plane of the interferometer (counted by convention from North through East so that North is $\theta_0 = 0$ and East is $\theta_0 = \pi/2$). By also introducing the magnitude $v = v(t)$ of the *projection* of the full $\mathbf{V}$, such that

$$v_x(t) = v(t)\sin\theta_0(t) \qquad\qquad v_y(t) = v(t)\cos\theta_0(t) \tag{23}$$

we obtain the theoretical relations [5]

$$A(t) = \frac{1}{2}|\mathcal{B}|\frac{v^2(t)}{c^2} \tag{24}$$

and

$$C(t) = \frac{1}{2}|\mathcal{B}|\ \frac{v_y^2(t) - v_x^2(t)}{c^2} \qquad\qquad S(t) = \frac{1}{2}|\mathcal{B}|\ \frac{2v_x(t)v_y(t)}{c^2} \tag{25}$$

In the forthcoming section we shall produce a numerical simulation by assuming a model of turbulent flow for the velocity components $v_x(t)$ and $v_y(t)$.

4. Numerical simulation of a physical, stochastic component

Before trying to simulate a physical stochastic component of the signal, to obtain the correct normalization, we should first subtract from the existing data the known spurious effects. To obtain a precise statistical indicator we shall consider the root square of the Allan variance (RAV) for an integration time $\tau \sim 1$ second which we'll take as our definition of instantaneous signal. In fact, our model predicts typical frequency shifts of about 1 Hz so that it only makes sense to compare with sequences of data collected at time steps of 1 second or larger. The RAV describes the time dependence of an arbitrary function $z = z(t)$ which can be sampled over time intervals of length τ. In this case, by defining

$$\overline{z}(t_i;\tau) = \frac{1}{\tau}\int_{t_i}^{t_i+\tau} dt\ z(t) \equiv \overline{z}_i \tag{26}$$

one generates a $\tau-$dependent distribution of $\overline{z}_i$ values. In a large time interval $\Lambda = M\tau$, the RAV is then defined as

$$\mathrm{RAV}(\tau) = \sqrt{\sigma^2(z,\tau)} \tag{27}$$

where

$$\sigma^2(z,\tau) = \frac{1}{2M}\sum_{i=1}^{M}(\overline{z}_i - \overline{z}_{i+1})^2 \tag{28}$$

Now, for the non-rotating set up, the RAV of the frequency shift for $\tau \sim 1$ second was determined [9] to be 0.8 Hz ($2.8 \cdot 10^{-15}$ in dimensionless units) [4] and found much larger than the corresponding disturbances in the individual resonators (typically about 0.02-0.03 Hz). The only exception is the possible effect of thermal noise in the mirrors of the optical resonators. This particular component, which should be independent of the integration time, on the basis of the results of ref.[30], was estimated in ref.[31] to be about $1.15 \cdot 10^{-15}$ in dimensionless units. Therefore, for a laser frequency $\nu_0 = 2.82 \cdot 10^{14}$ Hz, we would expect RAV(thermal-noise)$\sim$ 0.32 Hz. It is questionable how to subtract this effect from the full measured value 0.8 Hz. One might argue that, if the physical signal has also a stochastic nature, one should subtract in quadrature. This would give

$$\mathrm{RAV}(\mathrm{physical}, \tau \sim 1\ \mathrm{second}) = \sqrt{(0.8)^2 - (0.32)^2} \sim 0.73\ \mathrm{Hz} \tag{29}$$

Instead, we shall adopt the more conservative attitude of subtracting linearly, i.e.

$$\mathrm{RAV}(\mathrm{physical}, \tau \sim 1\ \mathrm{second}) = 0.8\ \mathrm{Hz} - 0.32\ \mathrm{Hz} = 0.48\ \mathrm{Hz} \tag{30}$$

or $1.7 \cdot 10^{-15}$ in dimensionless units. Since for a symmetric non-rotating set-up the physical signal is simply $2C(t)\nu_0$, we conclude that there is a potentially important contribution to $C(t)$ which corresponds to a stochastic signal with an Allan variance of about $8.5 \cdot 10^{-16}$ for $\tau \sim 1$ second. This value will be the basic input for our simulation.

Let us now return to Eqs.(25) and assume for the velocity components $v_x(t)$ and $v_y(t)$ a toy model of turbulent flow. To this end, we shall follow ref.[32] where velocity flows, in

[4] We tried to obtain an analogous indication from the other experiment of ref.[27]. However, for $\tau \sim 1$ second, it is not so easy to determine the value of the RAV. In fact, by inspection of their figure 2, in the narrow range from $\tau = 0.8$ seconds to $\tau = 1$ second, the data for the non-rotating set-up (the red dots) exhibit a very steep, sizeable decrease from about $2.8 \cdot 10^{-15}$ down to $1.4 \cdot 10^{-15}$.

statistically isotropic and homogeneous 3-dimensional turbulence, are generated by unsteady random Fourier series. The perspective is that of an observer which moves in the turbulent fluid and wants to simulate the two components of the velocity in his x-y plane at a given fixed location in his laboratory. This leads to the general expressions

$$v_x(t) = \sum_{n=1}^{\infty} [x_n(1)\cos\omega_n t + x_n(2)\sin\omega_n t] \tag{31}$$

$$v_y(t) = \sum_{n=1}^{\infty} [y_n(1)\cos\omega_n t + y_n(2)\sin\omega_n t] \tag{32}$$

where $\omega_n = 2n\pi/T$, T being a time scale which represents a common period of all stochastic components. In our simulation we have fixed the typical value $T = T_{\rm day}$= 24 hours. However, we have also checked with a few runs that the statistical distributions of the various quantities do not change appreciably if we vary T in the rather wide range 0.1 $T_{\rm day} \leq T \leq 10\ T_{\rm day}$.

The coefficients $x_n(i = 1, 2)$ and $y_n(i = 1, 2)$ are random variables with zero mean. They have the physical dimension of a velocity and we shall denote by $[-\tilde{v}, \tilde{v}]$ the relevant interval of these parameters. In terms of $\tilde{v}$ the quadratic mean values can be expressed as

$$\langle x_n^2(i = 1, 2)\rangle = \langle y_n^2(i = 1, 2)\rangle = \frac{\tilde{v}^2}{3\ n^{2\beta}} \tag{33}$$

for the uniform probability model (within the interval $[-\tilde{v}, \tilde{v}]$) which we have chosen for our simulations. Finally, the exponent β controls the power spectrum of the fluctuating components. For our simulation, between the two values $\beta = 5/6$ and $\beta = 1$ reported in ref.[32], we have chosen $\beta = 1$ which corresponds to the point of view of an observer moving in the fluid.

Thus, within this simple model for the stochastic signal, $\tilde{v}$ is our only free parameter and will be fixed by imposing that the generated $C-$values give a RAV of $8.5 \cdot 10^{-16}$ for integration time $\tau = 1$ second. By taking into account the typical variation of the results, due to both the truncation of the Fourier modes and the dependence on the random sequence, this constraint gives a range $\tilde{v} \sim (332 \pm 8)$ km/s which, remarkably, has a definite counter part in the known Earth's cosmic motion with respect to the Cosmic Microwave Background (CMB). In fact, it coincides exactly with the daily average of the projection $\sqrt{\langle v^2\rangle} \sim 332$ km/s in the interferometer's plane for an apparatus at the latitude of Berlin-Düsseldorf. This can be checked by using the relation [5]

$$\langle v^2\rangle = V^2 \left(1 - \sin^2\gamma\cos^2\chi - \frac{1}{2}\cos^2\gamma\sin^2\chi\right) \tag{34}$$

and setting $V = 370$ km/s, angular declination $\gamma \sim -6$ degrees and co-latitude $\chi \sim 38$ degrees.

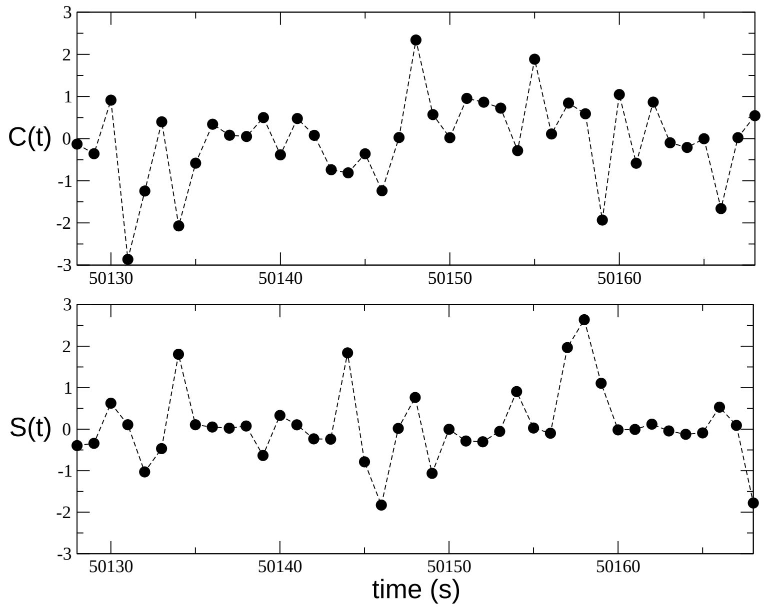

Figure 1: *A simulation of the instantaneous values of the C and S functions, in units* 10^{-15}, *as obtained from a typical sequence of 40 seconds* .

After these preliminaries, the results of our numerical simulation can be illustrated by starting from the building blocks of our scheme, namely the instantaneous values $C_i = C(t_i)$ and $S_i = S(t_i)$ of the C and $S-$functions. In Fig.1 we report a typical sequence of 40 values of these functions.

In terms of these basic quantities, one can construct a first type of averages over a time scale $\tau \equiv N$ seconds

$$\overline{C}(t_i; N) = \frac{1}{N} \sum_{n=i}^{i+N-1} C_n \qquad\qquad \overline{S}(t_i; N) = \frac{1}{N} \sum_{n=i}^{i+N-1} S_n \tag{35}$$

so that $\overline{C}(t_i; 1) = C_i$ and $\overline{S}(t_i; 1) = S_i$. This first type of averaging is essential to compare with the experimental data where the C and $S-$functions are always determined after averaging the basic instantaneous data over times τ in the typical range 40-400 seconds. With these auxiliary quantities, collected during a large time scale $\Lambda = M\tau$, one can form a statistical distribution and determine mean values

$$\langle C \rangle_\tau = \frac{1}{M} \sum_{i=1}^{M} \overline{C}(t_i; \tau) \qquad\qquad \langle S \rangle_\tau = \frac{1}{M} \sum_{i=1}^{M} \overline{S}(t_i; \tau) \tag{36}$$

and variances

$$\sigma^2_C(\tau) = \sum_{i=1}^{M} \frac{\left(\overline{C}(t_i; \tau) - \langle C \rangle_\tau\right)^2}{M-1} \qquad\qquad \sigma^2_S(\tau) = \sum_{i=1}^{M} \frac{\left(\overline{S}(t_i; \tau) - \langle S \rangle_\tau\right)^2}{M-1} \tag{37}$$

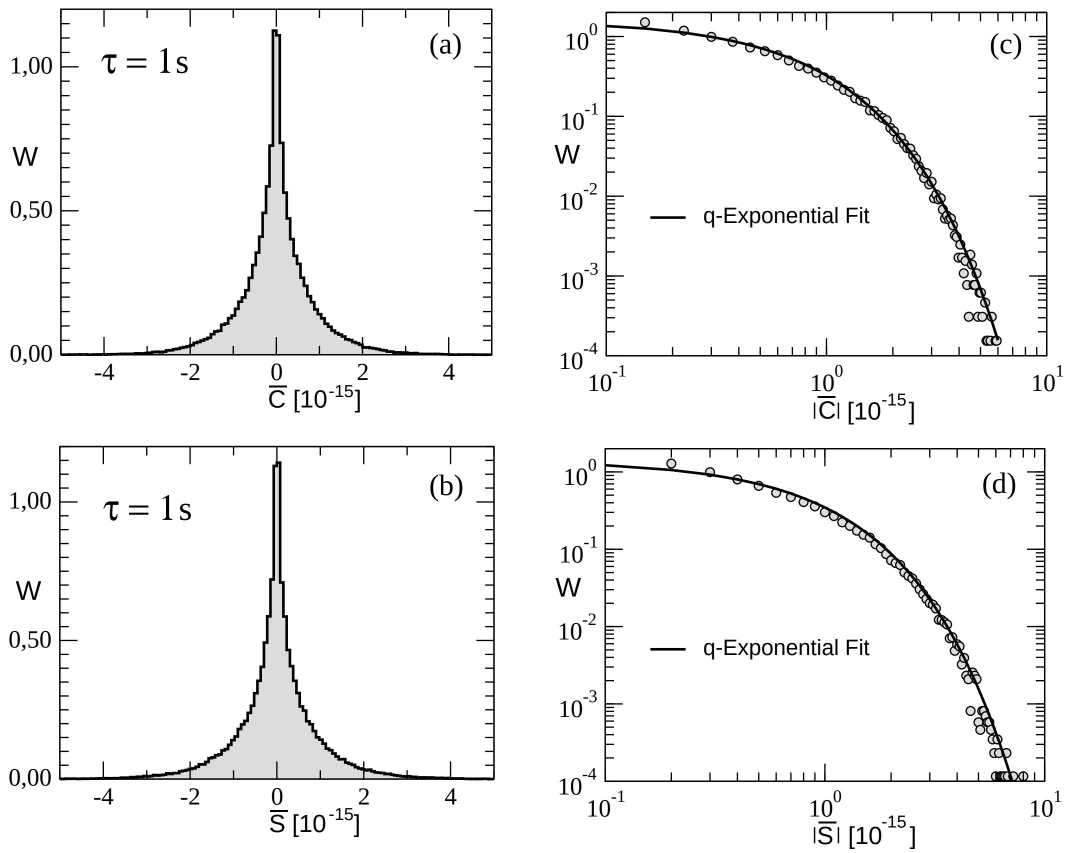

Figure 2: *We show, see (a) and (b), the histograms of the simulated* $\overline{C}$ *and* $\overline{S}$ *values, in units* 10^{-15}*, for* $\tau = 1$ *second. The vertical normalization is to a unit area. The mean values are* $\langle C \rangle_\tau = -1.1 \cdot 10^{-18}$*,* $\langle S \rangle_\tau = -1.9 \cdot 10^{-18}$ *and the standard deviations* $\sigma_C(\tau) = 8.5 \cdot 10^{-16}$*,* $\sigma_S(\tau) = 9.4 \cdot 10^{-16}$*. We also show, see (c) and (d), the corresponding plots in logarithmic scale and the fits with Eq.(38). The total statistics correspond to a time* $\Lambda = M\tau$ *=86400 seconds.*

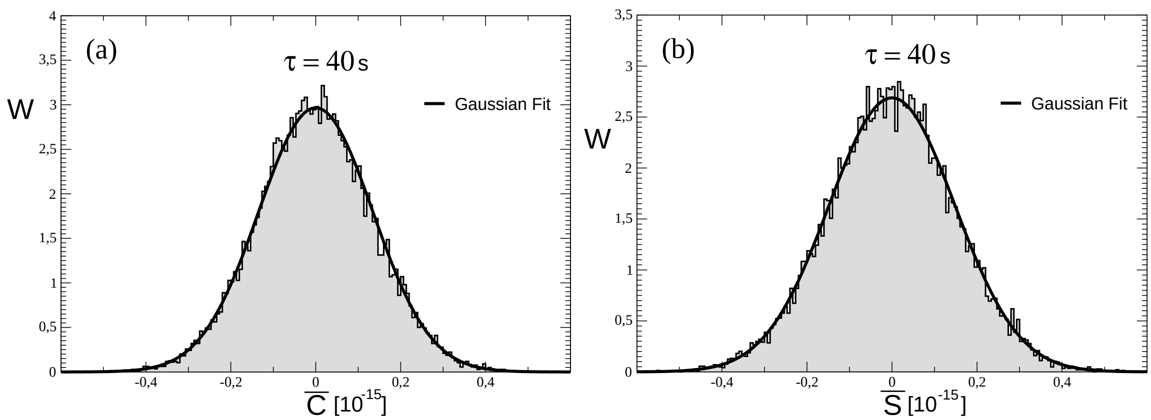

Figure 3: *The histograms of the simulated $\overline{C}$ and $\overline{S}$ values, in units 10^{-15}, and the corresponding gaussian fits for $\tau = 40$ seconds. The vertical normalization is to a unit area. The mean values are $\langle C\rangle_\tau = -9\cdot 10^{-19}$, $\langle S\rangle_\tau = -5\cdot 10^{-19}$ and the standard deviations $\sigma_C(\tau) = 1.34\cdot 10^{-16}$, $\sigma_S(\tau) = 1.48\cdot 10^{-16}$. The total statistics correspond to a time $\Lambda = M\tau$ =864000 seconds.*

We report in Fig.2, for $\tau = 1$ second, the distribution functions of the simulated $\overline{C}$ and $\overline{S}$ values (panels (a) and (b)). Notice that these distributions are clearly "fat-tailed" and very different from a Gaussian shape. This kind of behavior is characteristic of probability distributions in turbulent flow at small time scales (see e.g. [33, 34]). To better appreciate the deviation from Gaussian behavior, in panels (c) and (d) we plot the same data in a log−log scale. The resulting distributions are well fitted by the so-called $q-$exponential function [35]

$$y = A(1-(1-q)xB)^{1/(1-q)} \tag{38}$$

with an entropic index $q = 1.01$.

By starting to average the instantaneous values, the statistical distributions of the simulated $\overline{C}$ and $\overline{S}$ tend to assume a gaussian shape. This is already evident from about $\tau = 5-6$ seconds. In Fig. 3 we show the two distributions for $\tau = 40$ seconds.

As it might be expected, for all τ the statistical averages $\langle C\rangle_\tau$ and $\langle S\rangle_\tau$ are vanishingly small in units of the typical instantaneous signal $\mathcal{O}(10^{-15})$ and any non-zero value has to be considered as statistical fluctuation. The standard deviations, on the other hand, have definite values and exhibit a clear $1/\sqrt{\tau}$ trend so that, to good approximation, one can express

$$\sigma_C(\tau) \sim \frac{8.5\cdot 10^{-16}}{\sqrt{\tau(\text{sec})}} \qquad\qquad \sigma_S(\tau) \sim \frac{9.4\cdot 10^{-16}}{\sqrt{\tau(\text{sec})}} \tag{39}$$

By keeping $\tilde{v}$ fixed at 332 km/s, the above two values for $\tau = 1$ second have an uncertainty of about 5% which reflects the typical variation of the results due to both the truncation of the Fourier modes and the dependence on the random sequence.

Notice that our model predicts a monotonic decrease of the dispersion of the data by increasing the averaging time τ and, therefore, cannot reproduce the linear increase of the Allan variance which is seen, in all present room temperature experiments, above about $\tau = 100$ seconds. However, this might also be a spurious effect. For instance, in the cryogenic experiment of ref.[36] the Allan variance (in the quiet phase between two refillings of the tank of liquid helium) was found to exhibit a monotonic decrease up to about $\tau = 250$ seconds. It remains to be seen if this decreasing trend will be confirmed by the forthcoming generation of experiments with cryogenic sapphire resonators [17] that are expected to have a short-time stability of a few 10^{-18}. Thus it will be possible to obtain a precise check of our predictions. In particular, the typical instantaneous signal should be about 200−300 times larger than the experimental sensitivity and the distributions of the $\overline{C}(t_i;\tau)$ and $\overline{S}(t_i;\tau)$, for $\tau = 100$ seconds, should extend up to values which are still 20−30 times larger.

5. Summary and conclusions

As discussed in the Introduction, ether-drift experiments play a fundamental role for our understanding of relativity. In fact, they represent the only known experiments which, in principle, can distinguish Einstein's interpretation from the Lorentzian point of view with a preferred reference frame Σ. Up to now, the interpretation of the data has been based on a theoretical model where the hypothetical Σ is assumed to define a fixed direction in space. In this way all type of signals that are not synchronous with the Earth's rotation tend to be considered as spurious instrumental noise and no particular effort is made to understand if there could be genuine physical effects which do not fit within the adopted scheme.

However, there is a logical gap which has been missed so far. Even though the relevant Earth's cosmic motion corresponds to that indicated by the anisotropy of the CMB ($V \sim$370 km/s, angular declination $\gamma \sim -6$ degrees, and right ascension $\alpha \sim 168$ degrees) it might be difficult to detect these parameters in microscopic measurements of the speed of light performed in a laboratory. The link between the two concepts depends on the adopted model for the vacuum. The point of view adopted so far corresponds to consider the vacuum as some kind of fluid in a state of regular, laminar flow. In these conditions global and local informations on the flow coincide.

We believe that without fully understanding the nature of that substratum that we call physical vacuum, one should instead keep a more open mind. For instance the physical vacuum might be similar to a form of turbulent ether, an idea which is deep rooted in the basic foundational aspects of both quantum theory and relativity (see e.g. [11]) and finds additional motivations in those representations of the vacuum as a form of 'space-time foam' which indeed resembles a turbulent fluid [12, 13, 14, 15, 16]. In this case, global and local indications might be very different and there could be forms of random signals that have a genuine physical origin. For instance, by combining the point of view of ref.[5], where gravity is considered a long-wavelength phenomenon which emerges from a space-time which is fundamentally flat at very short distances, with the idea of a turbulent ether, one arrives to a stochastic signal of typical magnitude 10^{-15} which could fit very well with the present experimental data.

For this reason, after reviewing in Sects.2 and 3, the general theoretical framework and the basics of the present ether-drift experiments, we have presented in Sect.4 a simple numerical simulation of the possible effects that one might expect in a statistically isotropic and homogeneous turbulent flow. After subtracting the known forms of disturbances, we have found that the observed distribution of the data over a time scale of 1 second require a value $\sim$ 332 km/s of the scalar velocity parameter which characterizes the fluctuations. Remarkably, this has a definite counter part in the known Earth's cosmic motion with respect to the CMB. In fact, it corresponds exactly to the daily average of the projection of the Earth's velocity in the interferometer's plane for an apparatus placed at the latitude of Berlin-Düsseldorf. However, this correspondence with the global Earth's motion is only valid at the level of statistical distributions and is not detectable from the naive time dependence of the data. In conclusion, we predict a typical instantaneous signal which should be about 200$-$300 times larger that the sensitivity of the forthcoming generation of cryogenic experiments [17]. A confirmation of this prediction would represent unambiguous evidence for a physical ether-drift with non-trivial implications for our understanding of both gravity and relativity.

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